C.9.2 Independent section

1. The frequency response is given by

$$H(\omega) = 0.5(1 + e^{-i\omega}).$$

We need to find the angle of this as a function of ω. There are a number of ways to do this, but a clever way is to notice that

$$1 = e^{-i\omega/2}e^{i\omega/2}$$

and

$$e^{-i\omega} = e^{-i\omega/2}e^{-i\omega/2}$$

so

$$H(\omega) = 0.5e^{-i\omega/2}(e^{i\omega/2} + e^{-i\omega/2}) = 0.5e^{-i\omega/2}\cos(\omega/2).$$

Over the range of frequencies 0 to π radians/sample, $\cos(\omega/2)$ is non-negative, so the angle is

$$\angle H(\omega) = -\omega/2.$$

This is exactly what we found with the plot, linear phase with a slope of $-1/2$, indicating a delay of 1/2 sample.

2. The difference equation for the comb filter modified with the lowpass filter is

$$\forall\, n \in \textit{Integers}, \quad y(n) = x(n) + 0.5\alpha(y(n-N) + y(n-N-1)).$$

This defines an LTI system, so if the input is $x(n) = e^{j\omega n}$, then the output is $H(\omega)e^{i\omega n}$, where H is the frequency response. We can determine the frequency response using this fact by plugging this input and output into the above to get

$$H(\omega)e^{i\omega n} = e^{i\omega n} + 0.5\alpha(H(\omega)e^{i\omega(n-N)} + H(\omega)e^{i\omega(n-N-1)}).$$

This can be rewritten as

$$H(\omega)e^{i\omega n}(1 - 0.5\alpha e^{-i\omega N} - 0.5\alpha e^{-i\omega(N+1)}) = e^{i\omega n}.$$

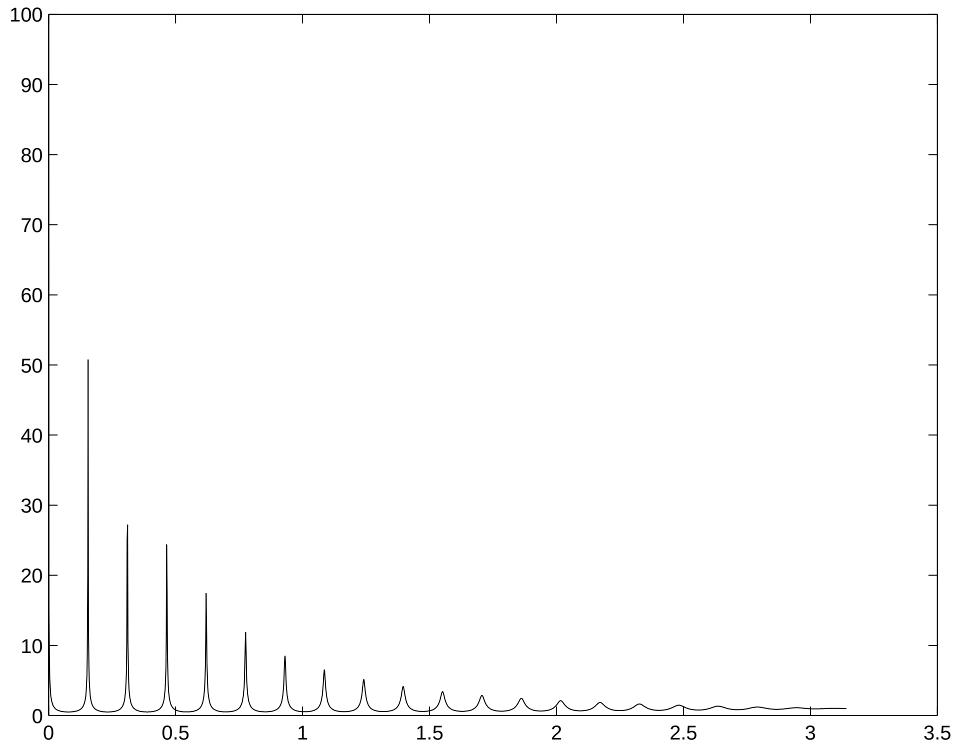
Eliminating $e^{i\omega n}$ and solving for $H(\omega)$ we get

$$H(\omega) = \frac{1}{1 - 0.5\alpha e^{-i\omega N} - 0.5\alpha e^{-i\omega(N+1)}}.$$

To plot the magnitude of this in the range 0 to π we can use the following Matlab commands:

```
omega = 0:pi/2000:pi;
alpha = 0.99;
N = 40;
H = 1./(1 - 0.5*alpha*exp(-i*omega*N) - 0.5*alpha*exp(-i*omega*(N+1)));
plot(omega, abs(H));
```

This results in the plot shown below:



Comparing against the comb filter of the previous lab, we see that the higher frequencies are attenuated relative to the comb filter.

Zooming into this plot, we find that the first peak is at 0.1555 radians/sample. To convert to Hz, we do the following calculation:

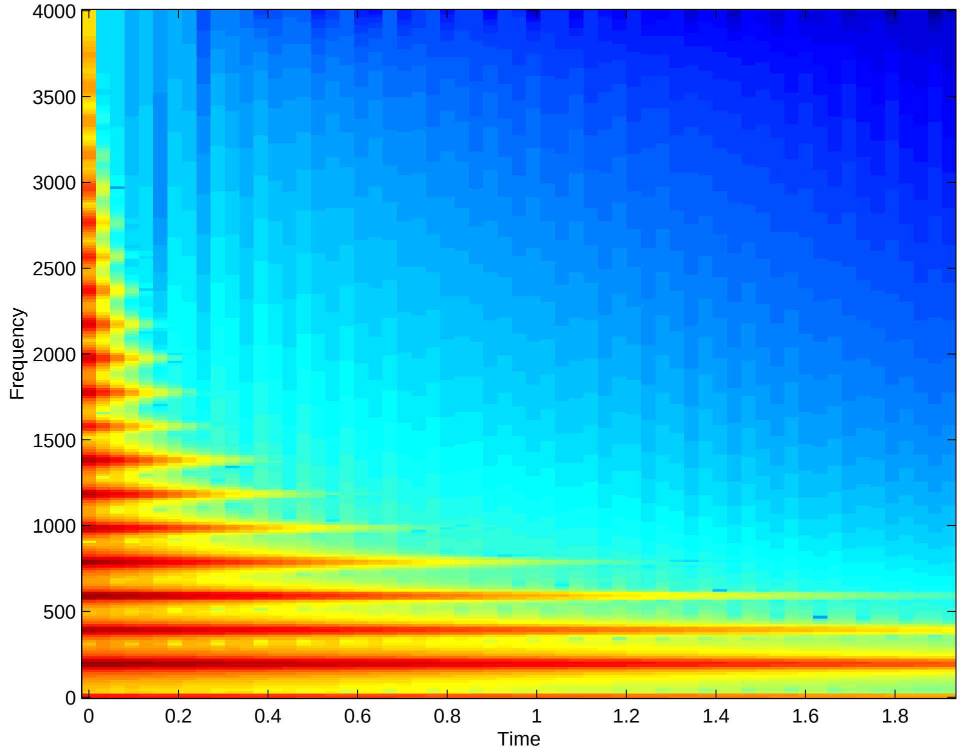
$$0.1555\text{[radians/sample]} \times 8000\text{[samples/second]}/2\pi\text{[radians/cycle]} = 198\text{Hz},$$

very close to the predicted value of 197.5 Hz.

3. The following Matlab command does the job:

```
specgram(simout, 512, 8000)
```

It yields the following image:



This image shows that immediately after starting, the higher harmonics are present and strong, but as the sound progresses, the higher harmonics get weaker.

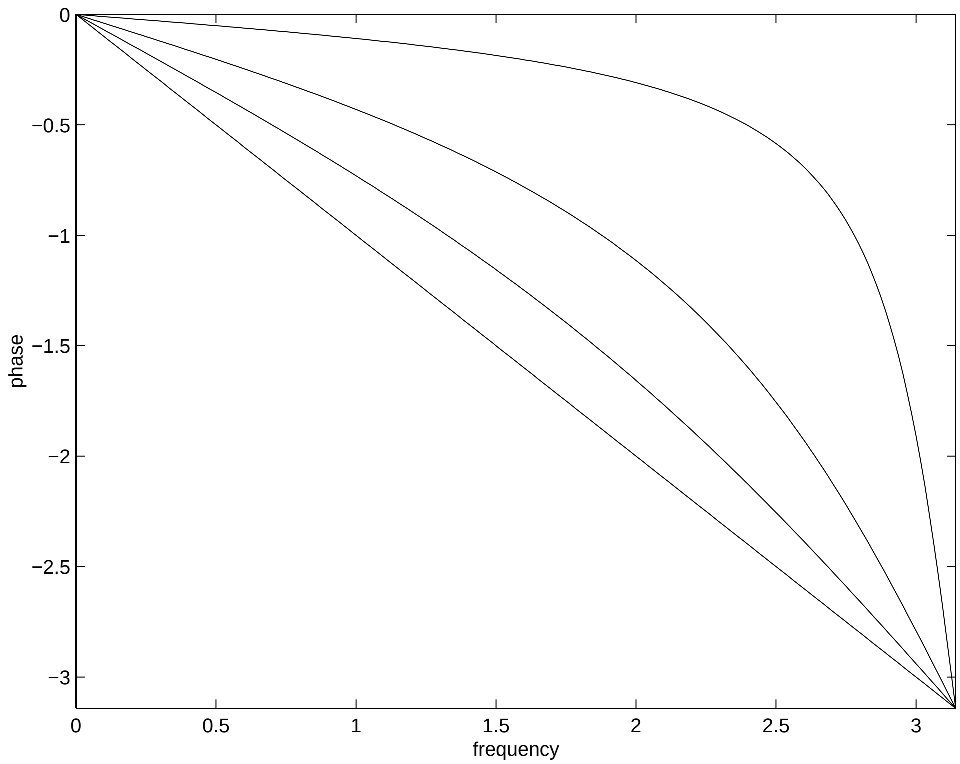
4. Solving for a in terms of d yields

$$a = \frac{1-d}{1+d}.$$

Thus, we can calculate the phase plots using the following Matlab code

```
omega = 0:pi/200:pi;
d = [0.1, 0.4, 0.7, 1.0];
a = (1-d)./(1+d);
for k=1:4
   H(k,:) = (a(k) + exp(-i*omega))./(1 + a(k)*exp(-i*omega));
end
plot(omega, angle(H));
axis([0, pi, -pi, 0]);
xlabel('frequency');
ylabel('phase');
```

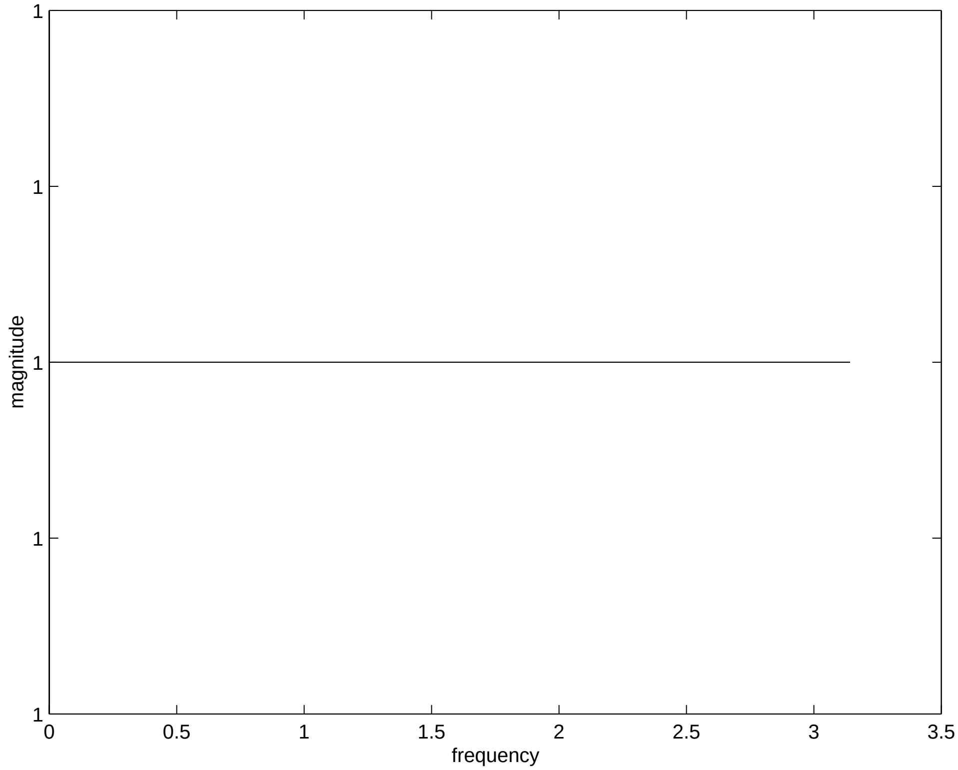
The result is shown below:



This gives the phase response of the allpass filter approximating a delay of 0.1, 0.4, 0.7, and 1.0 samples. Notice that at low frequencies the phase is roughly linear with slope $-d$. For the 1.0 sample delay, the phase is perfectly linear, which should not be surprising since a one sample delay is trivially achieved with discrete-time signals. To get the magnitude response, we just do

```
plot(omega, abs(H));
xlabel('frequency');
ylabel('magnitude');
```

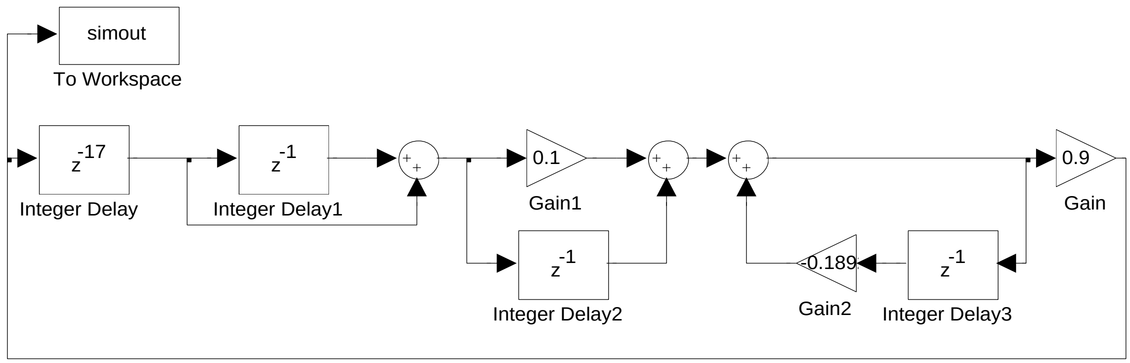
which results in the plot



5. We can rewrite the difference equation for the allpass filter as follows

$$\forall\, n \in \textit{Integers}, \quad y(n) = ax(n) + x(n-1) - ay(n-1)$$

which suggests the implementation shown below:



The delay needed from the allpass filter is a real number $0 < d < 1$ such that

$$8000/(N + 0.5 + d) = 440.$$

Thus

$$N + 0.5 + d = 8000/440 = 18.1818.$$

We choose $N = 17$ so $d = 18.1818 - 17.5 = 0.6818$. From part 4 above we know that

$$a = \frac{1-d}{1+d} = 0.1892.$$

The gain block in the above figure with label “0.1” has the gain parameter set to 0.1892, and the one with label “0.189” has gain -0.1892. Note also that the bulk delay has its parameter set to 17 rather than 40. Also, the feedback gain has been increased to 0.999 rather than 0.99 to get a longer decay of the sound. The result is a good A-440.