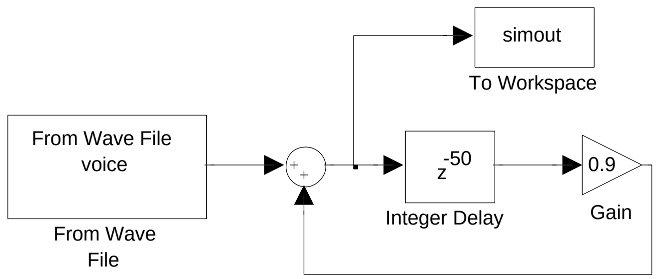

Figure C.21: A Simulink realization of the LCCDE in (**??**).

C.8 Comb Filters Solution

C.8.1 In-lab section

1. S_2 is a system with input y: *Ints* $\to$ *Reals* and output z: *Ints* $\to$ *Reals* given by

$$\forall\, n \in \textit{Ints}, \quad z(n) = \alpha y(n - N).$$

   Then

$$y(n) = x(n) + z(n).$$

   The LCCDE describes a system where the output is delayed, scaled, and feb back.

2. This system can be implemented in Simulink as shown in figure C.21. In figure C.21, $\alpha = 0.9$ and and $N = 50$.

   With $N = 2000$, the effect is an echo or reverberation. With $N = 50$, the effect is very different. The sound is similar to that you would hear speaking in round, reverberant chamber, such as a sewer pipe. The feedback in the LCCDE describes reflection of a sound. In a round reverberant chamber one meter in diameter, the sound bounces back and forth with the same amount of delay each time, and each time it is reflected, some energy is lost (hence $\alpha < 1$. The time that it takes to traverse one meter twice is approximately the same as a delay of 50 samples, when the sample rate is 8 kHz, as explained in the hint.

   When $\alpha > 1$, the system becomes unstable. Physically, this would correspond to the sound being amplified on each reflection. A plot of the output with $N = 50$ and $\alpha = 1.1$ is shown in figure C.22, which was created using the following Matlab commands:

```
plot([0:1/8000:4], simout)
xlabel('time'); ylabel('amplitude')
```

   When $\alpha = 0$, the output is identical to the input. When $\alpha = 1$, the system is marginally stable, and the reflections are not attenuated. They echo forever.

3. A plot of the impulse response with $N = 40$ and $\alpha = 0.99$ is shown at the top in figure C.23. A closeup of the first 0.05 seconds is shown at the bottom. These were created using the following Matlab commands:

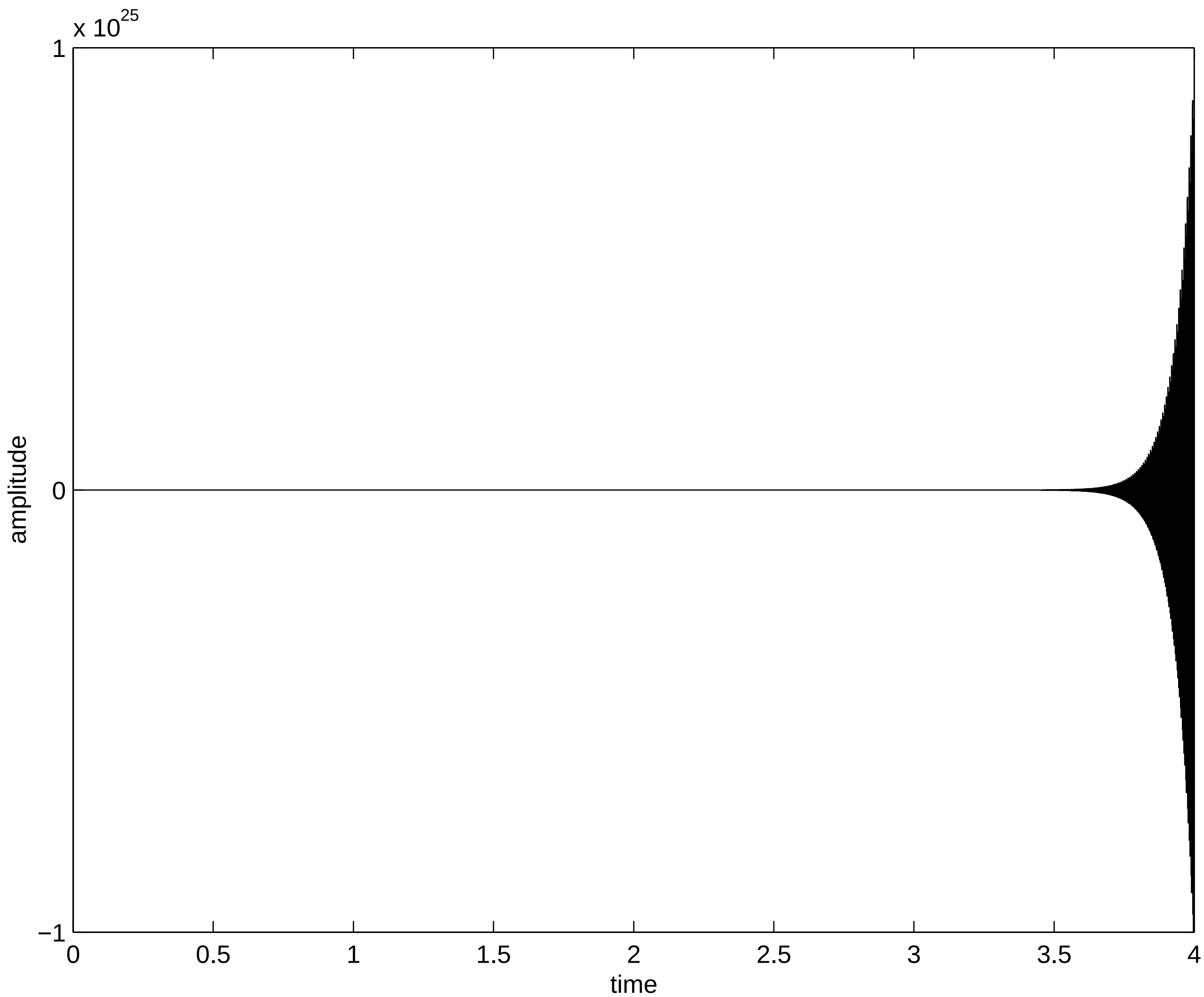

Figure C.22: Output of unstable feedback system.

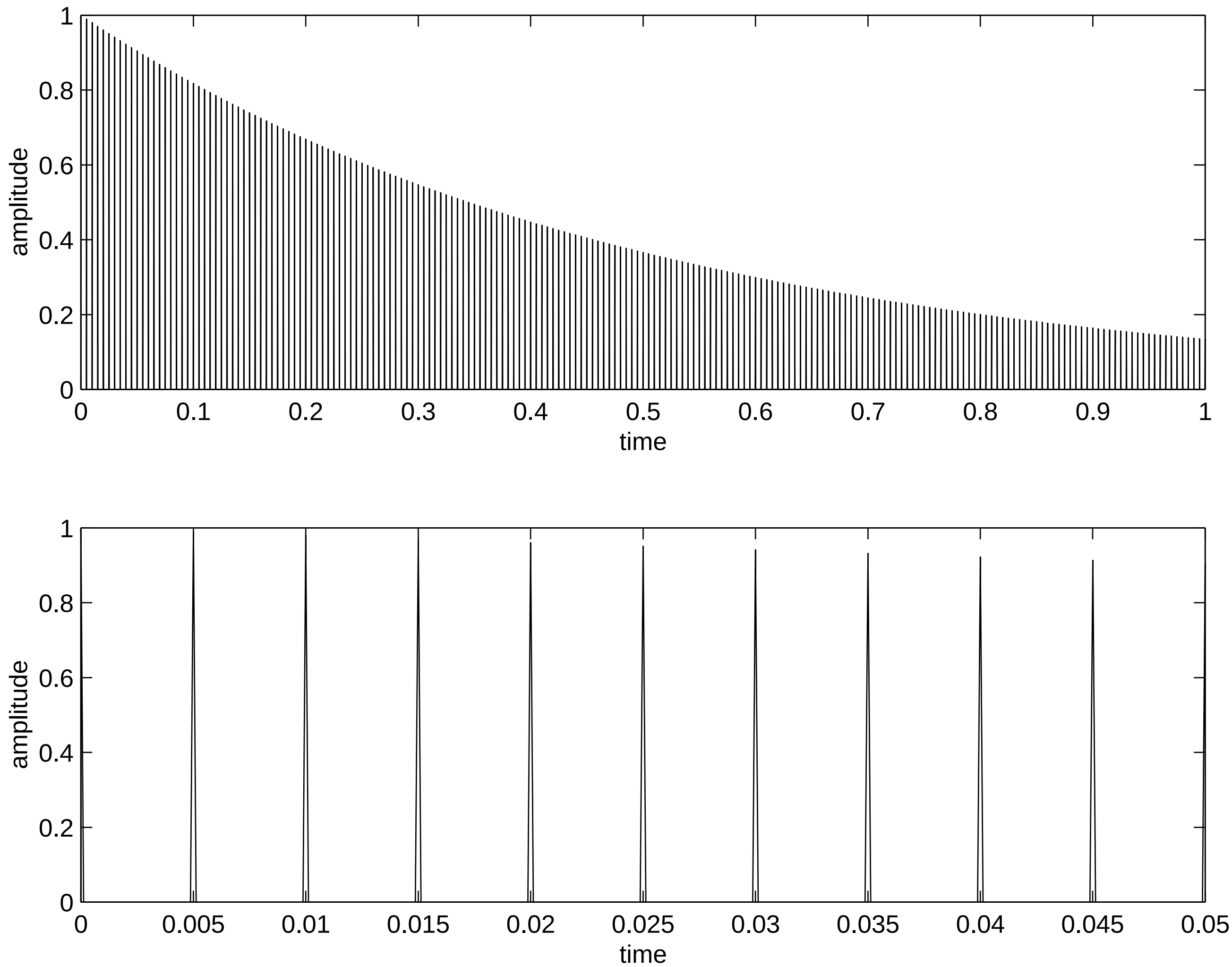

Figure C.23: Impulse response with $N = 40$ and $\alpha = 0.99$.

```
subplot(2,1,1); plot([0:1/8000:1], simout)
xlabel('time'); ylabel('amplitude')
subplot(2,1,2); plot([0:1/8000:0.05], simout(1:401))
axis([0, 0.05, 0, 1]);
xlabel('time'); ylabel('amplitude')
```

Note from the closeup that the impulse response is roughly periodic over short intervals with a period of 0.005 seconds, which corresponds to a fundamental frequency of 200 Hz. This is the tone of the sound we hear, although it is not a musical note (it does not align with any of the frequencies in the musical scale). We will address the problem of getting musical notes in the next lab.

4. The model with random initial values in the delay line is shown in figure C.24. The sound is indeed richer.