Figure C.16: Spectrogram and waveform for a train whistle.

C.7.2   Independent section

1. One realization of the reconstruction function looks like this:

```
function x = reconstruct(magnitude, phase)
% RECONSTRUCT - Given a vector of magnitudes and a vector
% of phases, construct a signal that has these magnitudes
% and phases as its discrete Fourier series coefficients.
% The arguments are assumed to have odd length, p/2 + 1,
% and the returned vector will have length p.

p = 2*(length(magnitude)-1);
n = 1:p;
x = zeros(1, p);
for k=1:length(magnitude)
    frequency = (k-1)/p;
    x = x + magnitude(k)*cos(2*pi*frequency*(n-1) + phase(k));
end
```

Notice that we can avoid the for loop, but only at the expense of using a rather large amount of memory to store a large array (8000 by 4000 in this case).
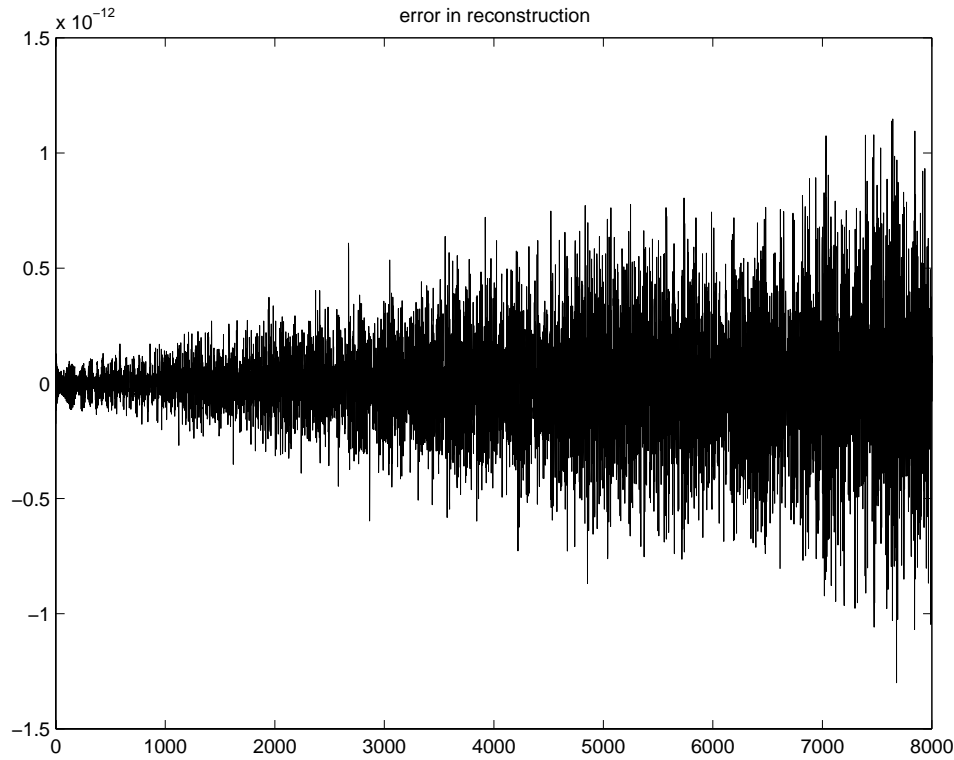
Figure C.17: The reconstruction error from Fourier series coefficients.

The following code calculates the discrete Fourier series coefficients reconstructs the original from them, and plots the error in the reconstruction. The result is shown in figure C.17. Notice that the error is mostly smaller than 10^{-12}.

```
t = [0:1/8000:1-1/8000];
y = sin(2*pi*800*(t.*t));
[A, phi] = fourierSeries(y);
yp = reconstruct(A, phi);
error = y - yp;
plot(error);
title('error in reconstruction');
```

2. From results in the specified box,

$$
\begin{aligned}
2\cos(\omega_c t)\cos(\omega_\Delta t) &= (e^{i\omega_c t} + e^{-i\omega_c t})(e^{i\omega_\Delta t} + e^{-i\omega_\Delta t})/2 \\
&= (e^{i(\omega_c+\omega_\Delta)t} + e^{-i(\omega_c+\omega_\Delta)t} + e^{i(\omega_c-\omega_\Delta)t} + e^{-i(\omega_c-\omega_\Delta)t})/2 \\
&= \cos((\omega_c+\omega_\Delta)t) + \cos((\omega_c-\omega_\Delta)t).
\end{aligned}
$$

where the last equality follows from Euler's relation.

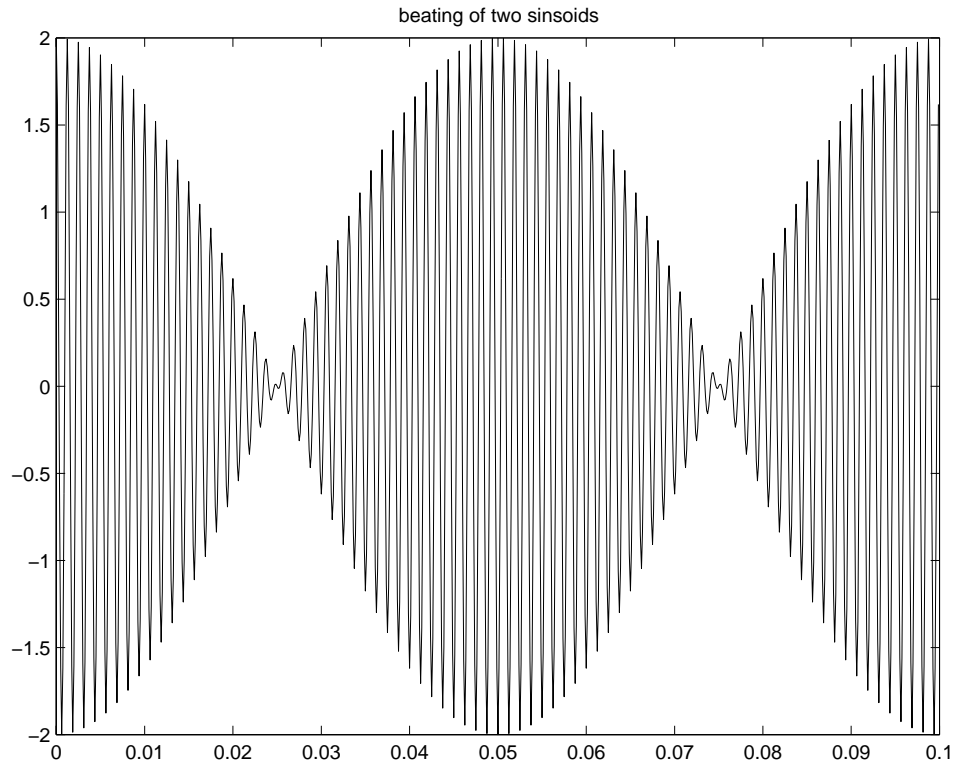3. The sum of the two sinusoids is generated by:

Figure C.18: Beating of two closely spaced sinusoids.

```
t = [0:1/8000:1-1/8000];
x = cos(2*pi*790*t) + cos(2*pi*810*t);
soundsc(x);
plot(t(1:800), x(1:800));
```

The resulting sound is more like a warble than a pair of tones. The plot of the first 1/10 second in figure C.18 explains why. It shows that the sum results in what looks like a sinusoid of one frequency with an envelope determined by another sinusoid.

To apply the identity in part 2, let $\omega_c = 800 \times 2\pi$ and $\omega_\Delta = 10 \times 2\pi$. Then we see that the sum of two sinusoids at 790 and 810 Hz is equivalent to the product of two sinusoids at 800 and 10 Hz. Indeed, the plot bears this out. The 10 Hz sinusoid imposes the envelope, and we hear the varying amplitude as warble.

4. The period will be the reciprocal of the greatest common divisor of 790 and 810, which is 1/10. This is evident in figure C.18. The fundamental frequency is therefore 10 Hz. The DFS coefficients can be calculated and plotted as follows:

```
[A, phi] = fourierSeries(x);
plot([0:4000], A);
```

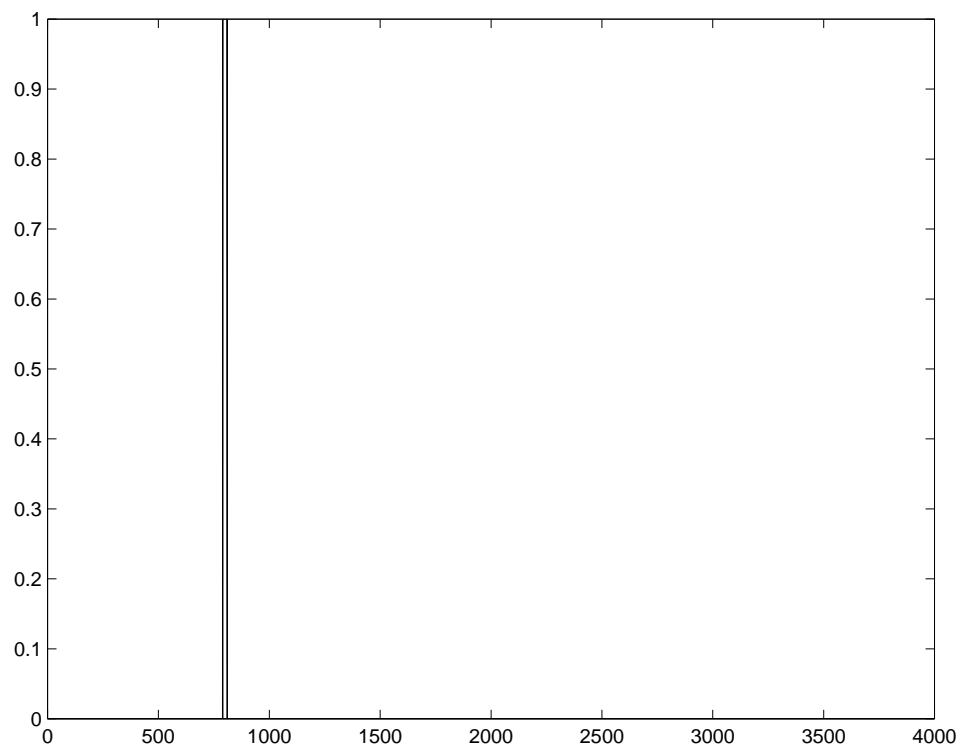The result is shown in figure C.19, which shows two very closely spaced peaks.

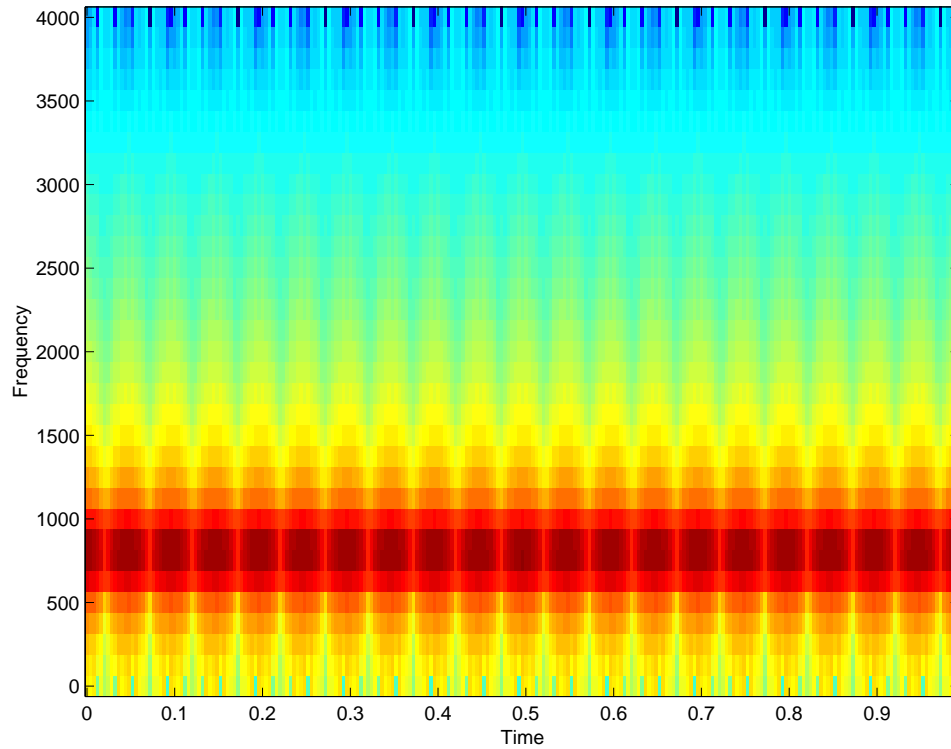Figure C.19: DFS of two closely spaced sinusoids.

Figure C.20: Spectrogram of two closely spaced sinusoids.

The spectrogram can be created as follows:

```
specgram(x,64,8000);
```

which results in the image shown in figure C.20. This image better reflects perception, because we hear warbling, not two steady tones.