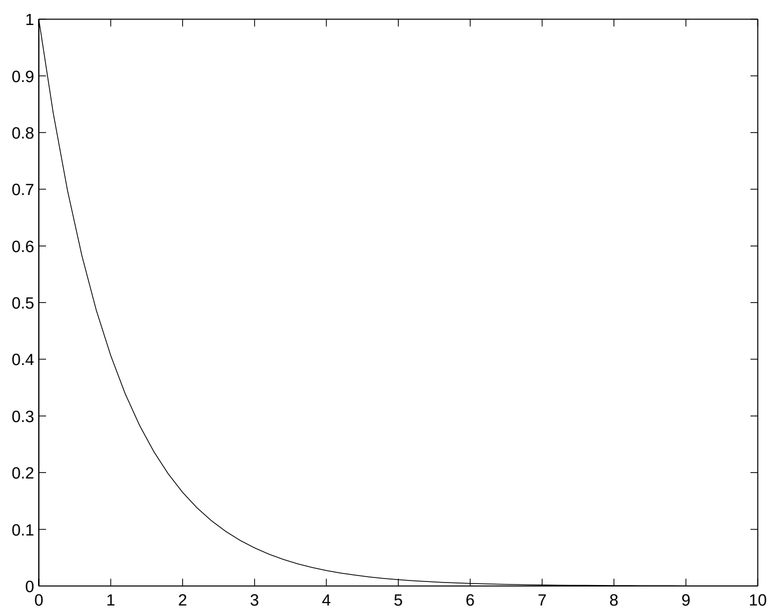
C.6 Differential Equations Solutions

C.6.1 In-lab section

1.

$$a = -0.9.$$

2. The result of the run is shown below:



Note that in a major oversight, Simulink does not provide a mechanism for exporting plots created by the Scope block to word processors. The above figure was generated by the very awkward mechanism of saving the data to the Matlab workspace, plotting it in Matlab, and then exporting the Matlab plot. Regrettably, even that is not simple to do. You have to be sure to export time to the workspace, itself not a totally trivial thing to do. We hope that The MathWorks will improve this situation someday.

3. The result is evidently a decaying exponential function. It must have the form, for all $t \in$ *Reals*$_+$,

$$z(t) = Ke^{\alpha t}$$

for some constants K and $alpha$. Since $z(0) = 1$, it must be true that $K = 1$, so

$$z(t) = e^{\alpha t}$$

Differentiating this, we find that

$$\dot{z}(t) = \alpha e^{\alpha t}$$

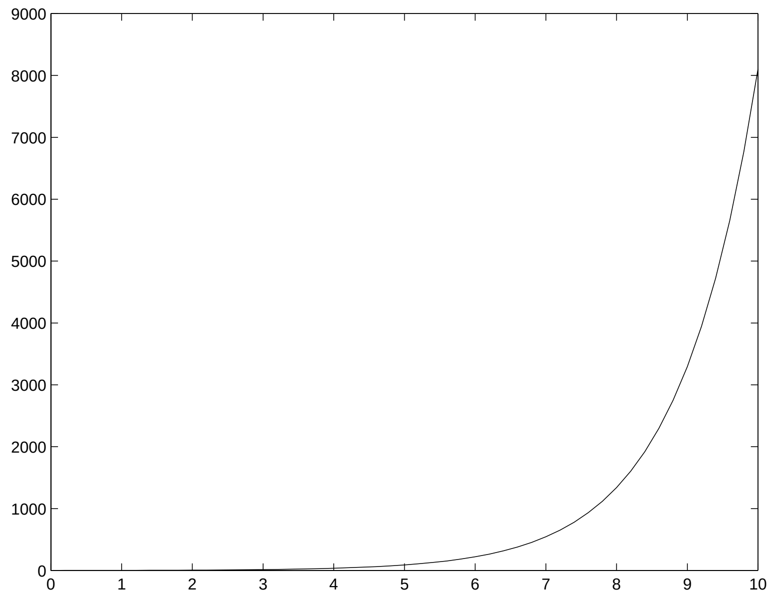
from which we can infer that $alpha = a = -0.9$, so

$$z(t) = e^{at}$$

and

$$\dot{z}(t) = ae^{at} = az(t).$$

4. With the gain set to 0.9 the result looks like this:



The output rapidly blows up. The system is clearly not stable. Once again,

$$z(t) = e^{at}$$

where now $a = 0.9$ This function grows without bound as t gets large.

5. The system is stable for any $a \leq 0$.