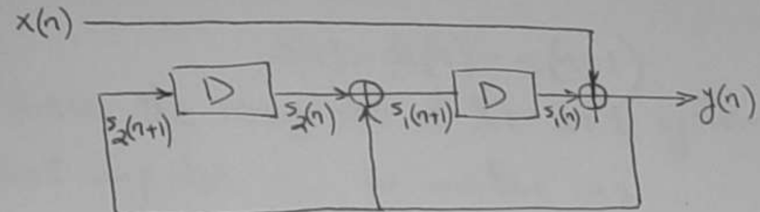
EECS20N (Fall 2005) HW7 Solutions

HW 7.1

$$y(n) = y(n-1) + y(n-2) + x(n)$$

a) The difference equation is second order ⟹ We can implement this system with a minimum of two delay elements. The delay-adder-gain block diagram is shown below:



b) From the diagram (labeling outputs of delay elements as state variables):

$s_1(n+1) = s_2(n) + y(n)$ (i)

$s_2(n+1) = y(n)$ (ii)

$y(n) = s_1(n) + x(n)$ (iii)

Inserting (iii) in each of (i) and (ii), we obtain:

$s_1(n+1) = s_1(n) + s_2(n) + x(n)$

$s_2(n+1) = s_1(n) + x(n)$

$y(n) = s_1(n) + x(n)$

which can be rewritten in standard state-space form as follows:

$$\begin{bmatrix} s_1(n+1) \\ s_2(n+1) \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}^{A} \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix} + \overbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}^{B} x(n)$$ State-Evolution Equation

$$y(n) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix} + \underbrace{1}_{D} \cdot x(n)$$ Output Equation

c) $h(n)$, $n \geq 0$ is the Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, . . . ($n=0$, $n=1$, $n=2$, $n=3$)

(every term starting with $n=2$ is the sum of the previous two terms)

HW 7.2

(a)

A suitable model is:

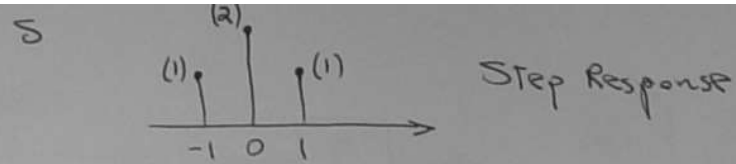
$$A = \begin{bmatrix} 0 & 0 & 0 & \alpha & \beta \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 0 & 0 & \alpha & \beta \end{bmatrix}, \quad D = \begin{bmatrix} 1 \end{bmatrix},$$

(b)

The only items in the state-space representation that change are B and D, which become

$$B = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad D = [0.5 \;\; 0.5].$$

HW 7.3



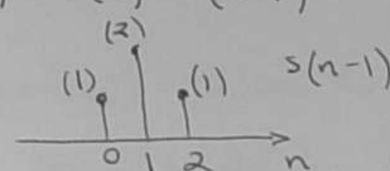
a) We note that the unit impulse function δ can be written as follows:

$$\delta(n) = u(n) - u(n-1)$$

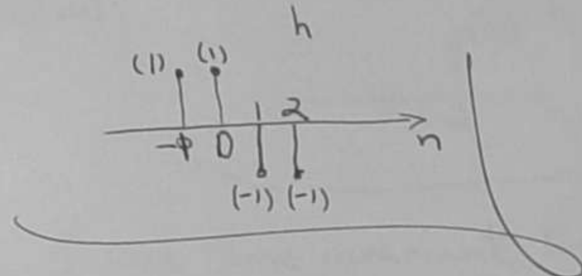
Hence, the response of the LTI system G to the unit impulse can be written as

$$h(n) = s(n) - s(n-1)$$

Noting that

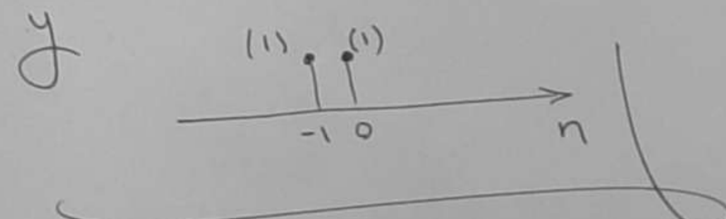


We have

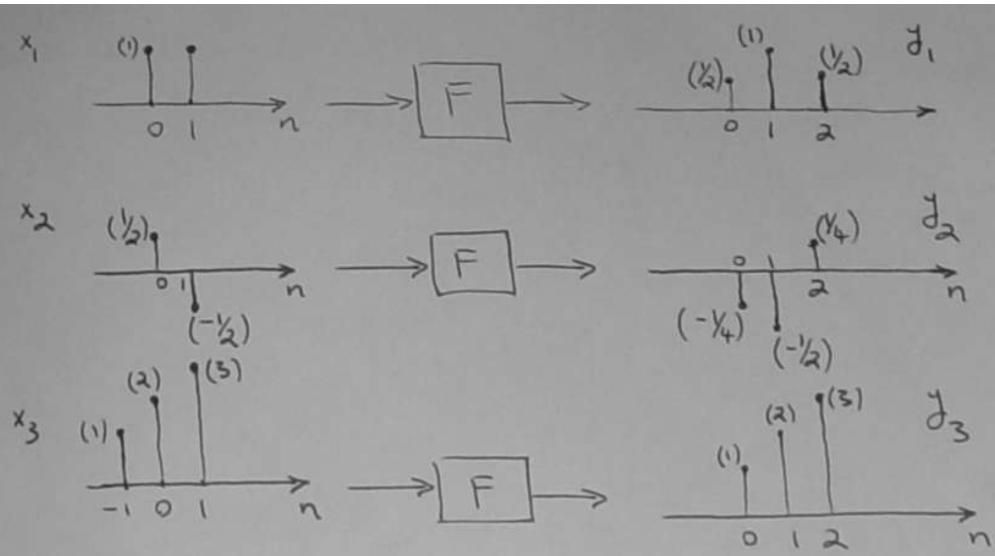


b) The response to $x(n) = \sum_{k=0}^{\infty} \delta(n-2k)$ is

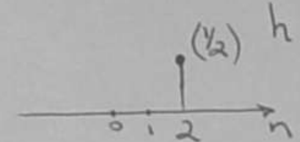
y such that $y(n) = \sum_{k=0}^{\infty} h(n-2k)$



HW 7.4



a) Note that $\delta = \frac{1}{2}(x_1 + 2x_2)$. Since the system is linear, the output is $h = \frac{1}{2}(y_1 + 2y_2)$. The plot of h is shown below:

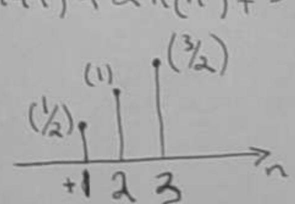


$h(n) = \frac{1}{2}\delta(n-2)$

b) If the system were time invariant, then it would be true that $y_3(n) = h(n+1) + 2h(n) + 3h(n-1)$ because

$$x_3(n) = \delta(n+1) + 2\delta(n) + 3\delta(n-1)$$

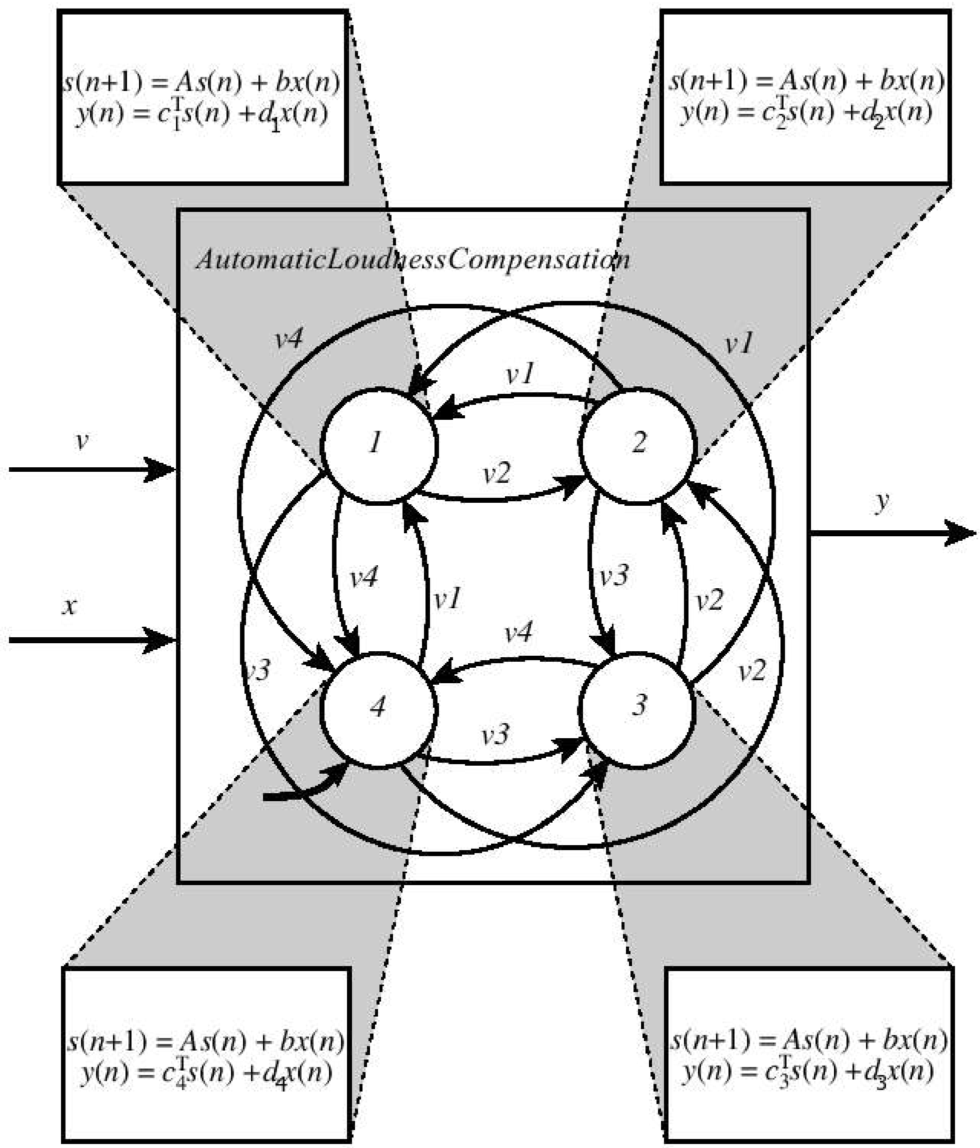
However, $h(n+1) + 2h(n) + 3h(n-1)$ is shown below:



Clearly, this is not equal to y_3. Hence, F cannot be time-invariant.

HW 7.5

The following hybrid system will do the job:



The inputs are two discrete-time signals, v representing the current volume setting, and x representing the audio signal. Each state i has a refinement that defines the state update and output as

$$\begin{aligned} \forall\, n \in \mathbb{Z}, \quad s(n+1) &= As(n) + bx(n) \\ y(n) &= c_i^T s(n) + d_i x(n). \end{aligned}$$

The guard on each transition into state i is a set $v_i \subset Inputs$ given by

$$v_i = \{(v(n), x(n), s(n), y(n)) \mid T_{i-1} \le v(n) < T_i\},$$

where $T_0, \cdots, T_5$ are thresholds governing the levels where the filtering switches. $T_0 = 0$ is the lowest level and $T_4 = \infty$ is the highest.

HW 7.6

(a) The system generates an event sequence

$$(1, 3, 4, 6, 7, 9, 10, \cdots)$$

at times

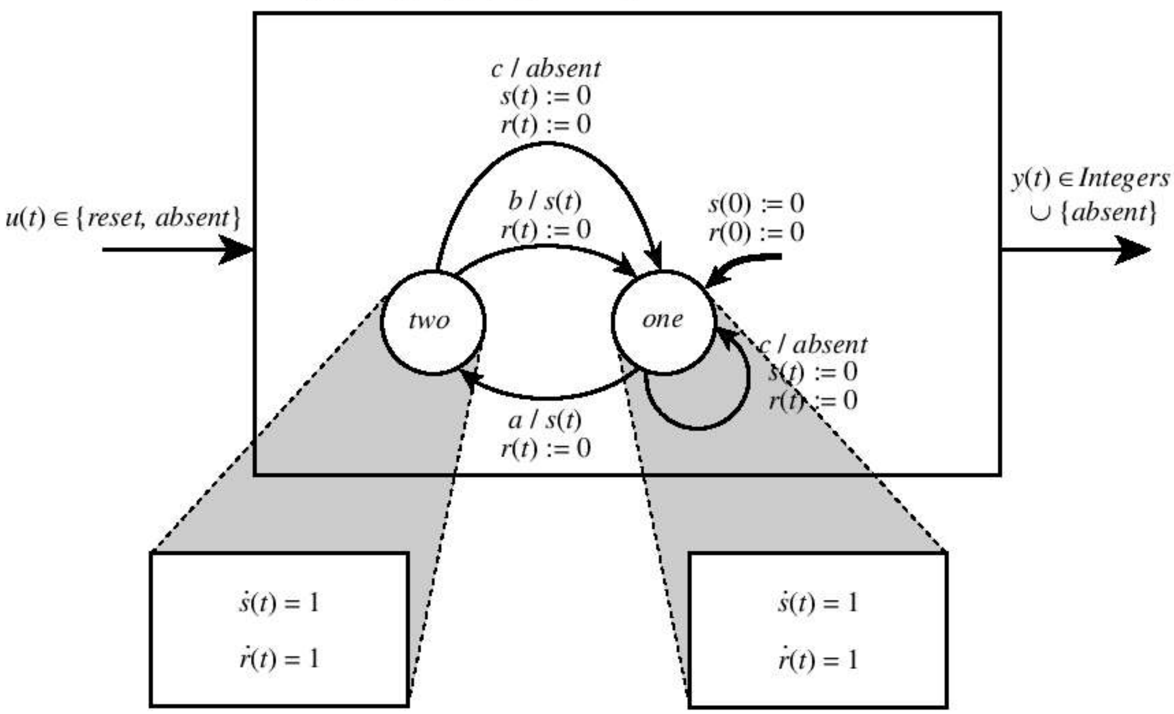
$$1, 3, 4, 6, 7, 9, 10, \cdots.$$

That is, the value of each output event is equal to the time at which it is produced, and the intervals between events alternate between one and two seconds. Precisely,

$$y(t) = \begin{cases} t & \text{if } t = 3k \text{ for some } k \in \mathbb{N} \\ t & \text{if } t = 3k+1 \text{ for some } k \in \mathbb{N}_0 \\ absent & \text{otherwise} \end{cases}$$

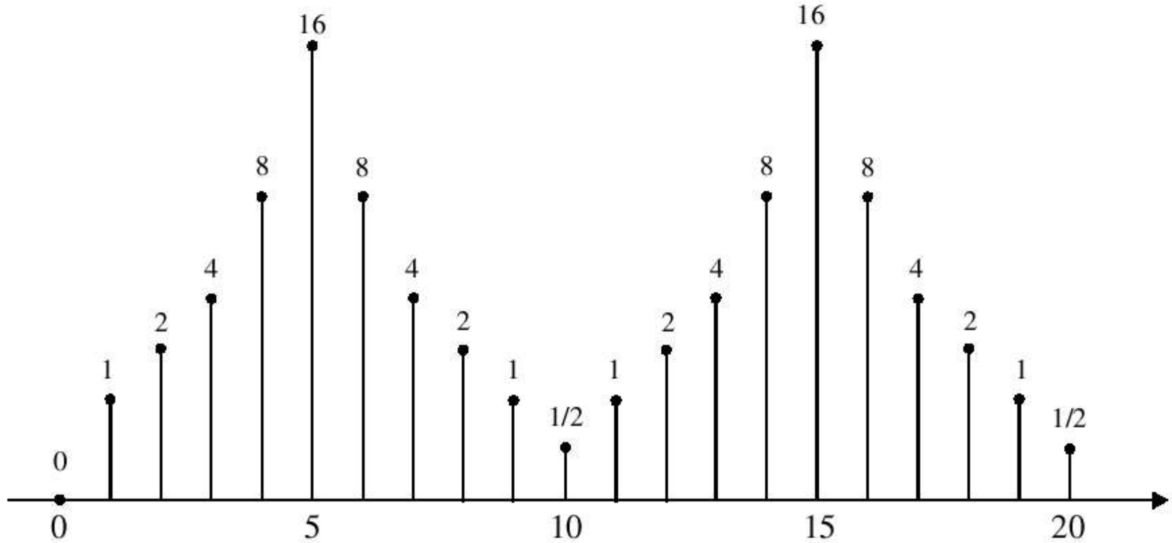
(b)

$$a = \{(r(t), s(t), u(t)) \mid r(t) = 1 \wedge u(t) = absent\}$$
$$b = \{(r(t), s(t), u(t)) \mid r(t) = 2 \wedge u(t) = absent\}$$
$$c = \{(r(t), s(t), u(t)) \mid u(t) = reset\}$$



HW 7.7

(a)



(b)

No, overall state $S = \{1, 2\} \times \text{Reals}$

$$\text{state} = (\text{mode}(n), r(n))$$

$\text{mode}(n)$	$r(n)$	$\text{mode}(n+1)$	$r(n+1)$	output $y(n)$
1	$\lvert r(n)\rvert \le 10$	1	$2r(n) + x(n)$	$r(n)$
1	$\lvert r(n)\rvert > 10$	2	$\frac{1}{2}r(n) + x(n)$	$r(n)$
2	$\lvert r(n)\rvert < 1$	1	$2r(n) + x(n)$	$r(n)$
2	$\lvert r(n)\rvert \ge 1$	2	$\frac{1}{2}r(n) + x(n)$	$r(n)$

(c)

$$\text{state}(n) = (\text{mode}(n),\ \text{time in mode 2},\ r(n))$$

$$S = \{1, 2\} \times \{0, 1, 2\} \times \text{Reals}$$