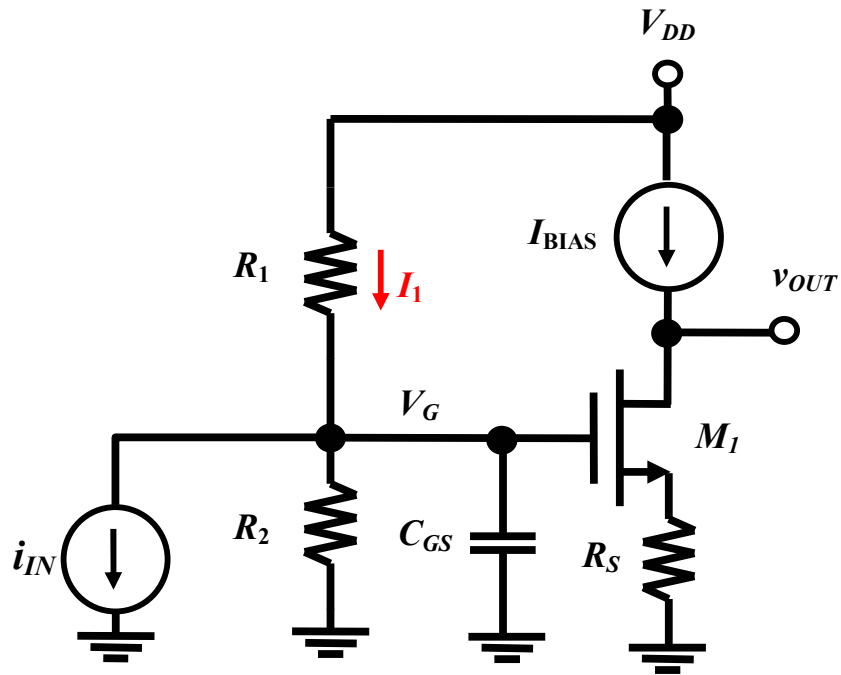


Name Good Student

Problem 4. MOSFET [30 points]



Parameters:

$$V_{DD} = 9V$$
$$I_{BIAS} = 2mA$$

$$R_1 = 3k\Omega$$
$$R_2 = 6k\Omega$$
$$R_S = 500\Omega$$

$$V_T(M_1) = 0.5V$$
$$\lambda(M_1) = 0.5V^{-1}$$
$$\mu_n C_{ox}(M_1) = 100 \text{ uA/V}^2$$
$$W/L(M_1) = 2$$

a. The voltage V_G considering $I_{IN} = 1\text{mA}$. Note that $i_{IN} = i_{in} + I_{IN}$ [5pts]

$$V_G = V_{DD} - I_1 \cdot R_1$$

$$I_1 = i_{IN} + \frac{V_G}{R_2}$$

$$V_G = 9V - (i_{IN} + \frac{V_G}{6k}) \cdot 3k$$

$$\frac{3}{2}V_G = 9V - i_{IN} \cdot 3k\Omega$$

$$\frac{3}{2}V_G = 9V - (i_{in} + 1mA) \cdot 3k\Omega$$

$$V_G = 4V - 2k \cdot i_{in}$$

$V_G = 4V$ if i_{in} is 0 for DC analysis

Mistake 1: Ignoring $I_{IN} = 1\text{mA}$, and just do a voltage divide. (-0.5 point)

Mistake 2: Plug in the wrong values for unknown reasons. (-0.5 point)

Mistake 3: Wrong sign when setting up the equation(+ or -) (-0.5 point)

b. Find the source voltage of M_1 . [5pts]

$$V_S = I_{BIAS} \cdot R_S = 1V$$

c. Find V_{DS} using the values from **a.** and **b.** What region is M1 operating in? [5pts]
Assume M1 is in saturation region.

$$I_{DS} = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

$$2mA = (100\mu A / V^2) \frac{2}{2} (3V - 0.5V)^2 (1 + 0.5V^{-1} \cdot V_{DS})$$

$$V_{DS} = 4.4V, V_{GS} = 3V, V_t = 0.5V$$

$$V_{DS} > V_{GS} - V_t \quad \text{and} \quad V_{GS} > V_t$$

Assumption was correct, and M1 is in Saturation Region.

Mistake 1: Wrong Vds (-0.5)
Mistake 2: No Vds but otherwise correct (-1)

d. Find the transfer function v_g / i_{in} . Note that i_{in} is a small signal without a DC component. [5pts]

$$V_g = -i_{in} (R_1 // R_2 // Z(C_{GS}))$$

$$\frac{V_g}{i_{in}} = - \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_{GS}\right)}$$

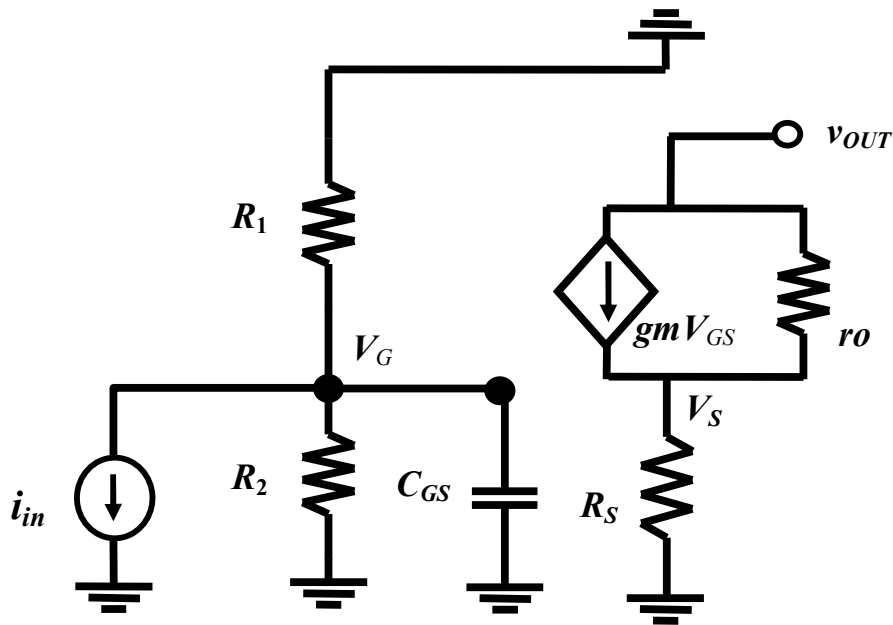
Common Mistake 1: Forgetting the negative sign (-0.5 point)

Common Mistake 2: Inverting the answer V_g / i_{in} because the student forgot how to calculate impedance in parallel (-0.5 point)

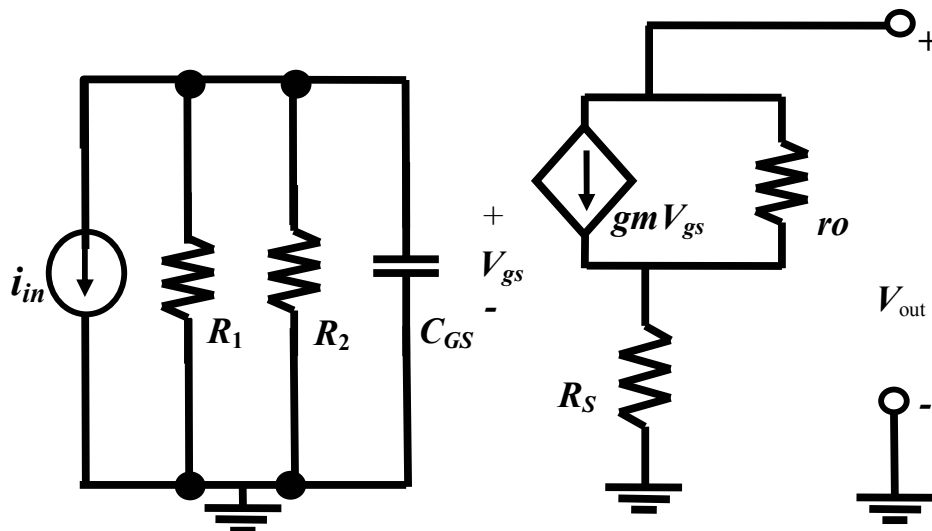
Common Mistake 3: Both Mistake 1 and Mistake 2. (-1 point)

- e. Sketch the small signal model of this configuration and find the transfer function v_{out}/i_{in} . [10 pts: 5pts for the small signal model, 5 pts for the calculations]

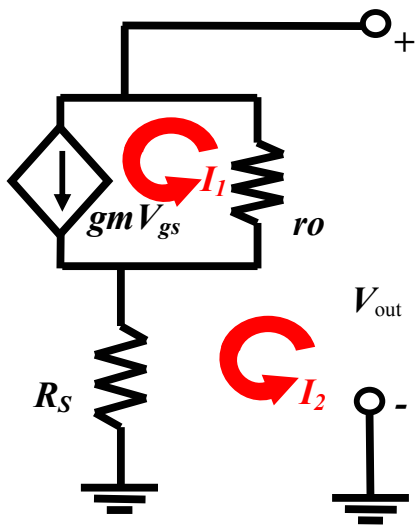
Step 1. Replace the circuit with all small circuit elements. That is eliminate all the DC (constant voltage or constant current) elements.



Step 2. which gives a small signal model of:



- Notice, the small signal circuit is a new circuit, the “big” signal V_G , V_S , and V_{OUT} are not here. Only small signal elements exist in a small signal model.
- C_{GS} is a misnomer. It is not the capacitor between the gate and source of M1.



$$\begin{cases} I_1 = gmV_{gs} \\ I_1 = \frac{V_S - V_{out}}{r_o} \\ V_{out} = (I_2 - I_1)r_o + I_2 \cdot R_S \\ I_2 = \frac{V_S}{R_S} \end{cases}$$

$$gm = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GSQ} - V_t) = 0.5m$$

$$r_o = \frac{1}{\lambda I_{DS}} = 1k$$

$$\begin{cases} I_1 = 0.5m (V_g - V_S) \\ I_1 = \frac{V_S - V_{out}}{1k} \\ v_{out} = (I_2 - I_1) \cdot 1k + I_2 \cdot 500 \\ I_2 = \frac{V_S}{500} \end{cases}$$

$$V_{out} = -1.375 V_g$$

$$\frac{V_{out}}{i_{in}} = 1.375 (R_1 // R_2 // Z(C_{GS}))$$

Common Mistake 1: The circuit said v_{OUT} , but this question asked for v_{out}/v_{in} . So the v_{out} is supposed to be small signal output only, without the DC bias.

Common Mistake 2: Wrong small signal circuit. Some assumed C_{GS} is infinite and shorted C_{GS} in the small signal circuit.

BONUS [2 points]: Find the output impedance of this configuration.

$$R_{out} = \frac{1}{gm} // r_o + R_S$$

$$R_{out} = r_o + R_S + gm \cdot R_S \cdot r_o$$