Notes about Small Signal Model

for EE 40 Intro to Microelectronic Circuits

1. Model the MOSFET Transistor

For a MOSFET transistor, there are NMOS and PMOS. The examples shown here would be for NMOS.

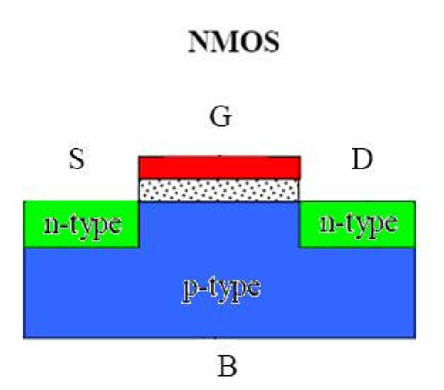


Figure 1. NMOS Transistor

2. Small Signal Current

$$i_{ds} = \underbrace{\frac{\partial I_{DS}}{\partial V_{GS}} \cdot v_{gs}}_{\text{Small change in source-drain current brought about by } v_{gs}} + \underbrace{\frac{\partial I_{DS}}{\partial V_{DS}} \cdot v_{ds}}_{\text{Small change in source-drain current brought about by } v_{ds}}$$

The partial derivatives have special names. More specifically.

$$\frac{\partial I_{DS}}{\partial V_{GS}} = gm \quad \text{and} \quad \frac{\partial I_{DS}}{\partial V_{DS}} = g_{ds} = \frac{1}{ro}$$

Hence, we can rewrite the equation as follows

$$i_{ds} = gm \cdot v_{gs} + g_{ds} \cdot v_{ds}$$

Notice we are ignoring Vbs here.

3. Small Signal Model

After determining the DC Bias Point, we are usually interested in the AC property of the circuit. Therefore, we need the small signal model to analyze the circuit. The simplified small signal model ignores the effect of Vbs and capacitors between each terminal.

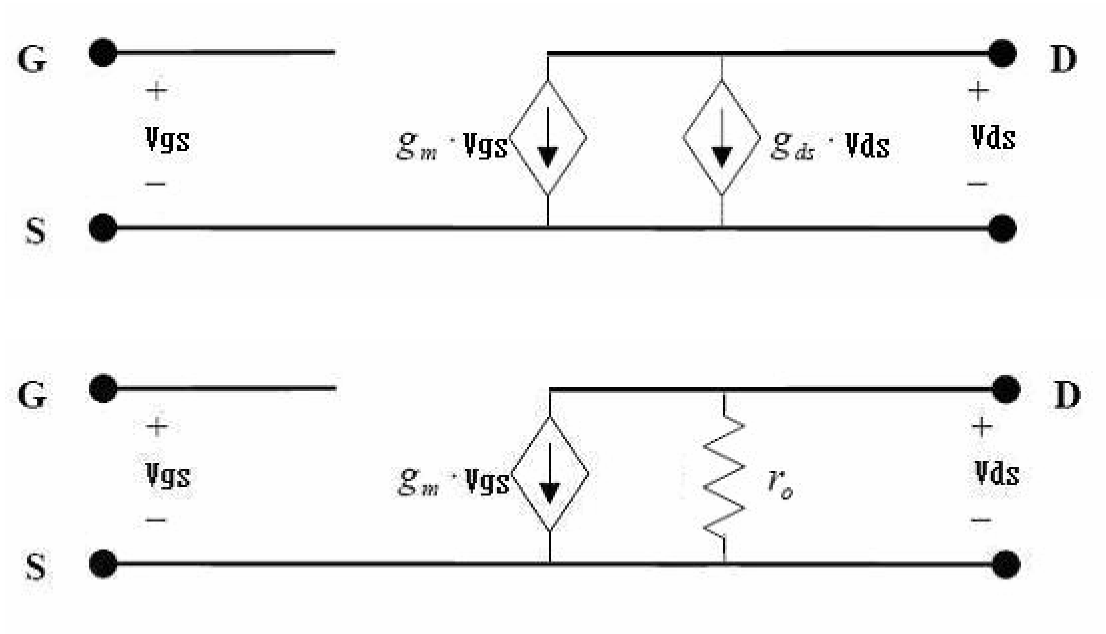


Figure 2. Simplified Small Signal Model

4. Turning the Circuit into Small Signal Model

In order to obtain the small signal circuit, we need to turn all large signal (DC) elements into its corresponding small signal elements. For example,

- The transistor would be replaced by the small signal model circuit
- Constant voltage and constant current would be eliminated from the circuit , and whether they should be an open circuit or a short circuit would depend on the situation.
- If, and only if, the capacitor has infinite capacitance, then you can short the capacitor.
- Only small signal (non-DC) elements exist in small signal model.

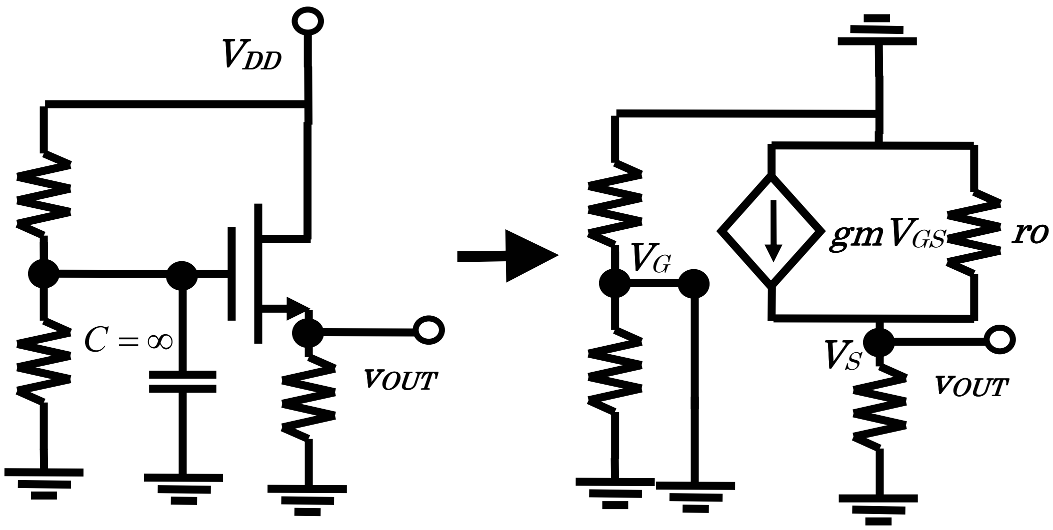


Figure 3. Large signal elements replaced by small signal elements. Notice the *infinite* capacitor is shorted, the VDD is grounded, and the transistor is replaced by the small signal circuit.

5. Working with Small Signal Model

   Usually the question would ask you to find the input and output resistance, the gm, the ro, the gain of the circuit, and then ask you to find the transfer function. In order to find these, you usually need to find the resistance "looking" into the terminals of the transistor. Here are three general rules for a NMOS transistor *with the gate and bulk tied to the small signal ground*, and we are "looking" into the gate, the drain, and the source for the resistances. Notice, if the gate is connected to a constant voltage, it is the same things as being connected to the small signal ground.

Rule #1 Gate Resistance

The resistance "looking" into the gate of a MOSFET transistor (NMOS or PMOS) with the gate being at small-signal ground is infinite because one is essentially looking into the gate capacitor.

$$r_{gate} = \infty \qquad (1)$$

Rule #2 Drain Resistance

The resistance "looking" into the drain of a MOSFET transistor (NMOS or PMOS) with the gate being at small-signal ground is given by the following expression (See Figure 4). Notice we are ignoring Vbs here.

$$r_{drain} = r_o + r_{source} + gm \cdot r_o \cdot r_{source} \qquad (2)$$

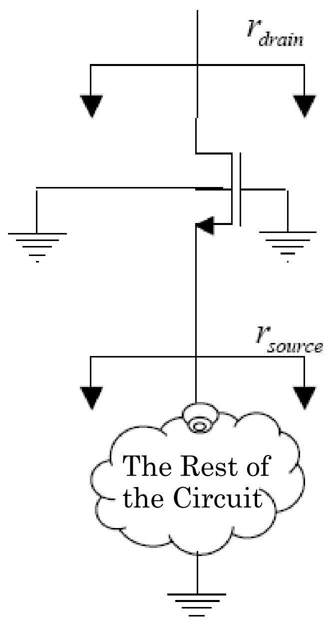


Figure 4

Rule #3 Source Resistance

The resistance “looking” into the source of a MOSFET transistor (NMOS or PMOS) with the gate being at small-signal ground is given by the following expression (See Figure 5). Notice we are ignoring Vbs here.

$$r_{source} = \frac{r_o + r_{drain}}{gm \cdot r_o + 1} \approx \frac{r_o + r_{drain}}{gm \cdot r_o} \quad (3)$$

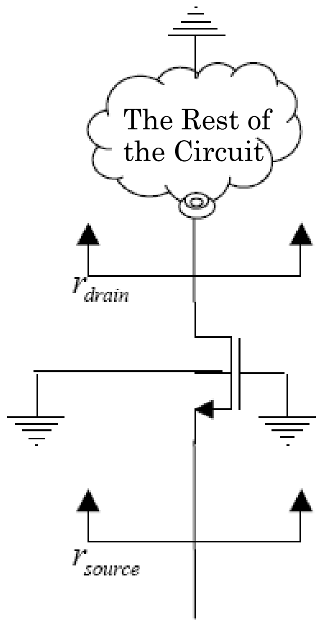


Figure 5

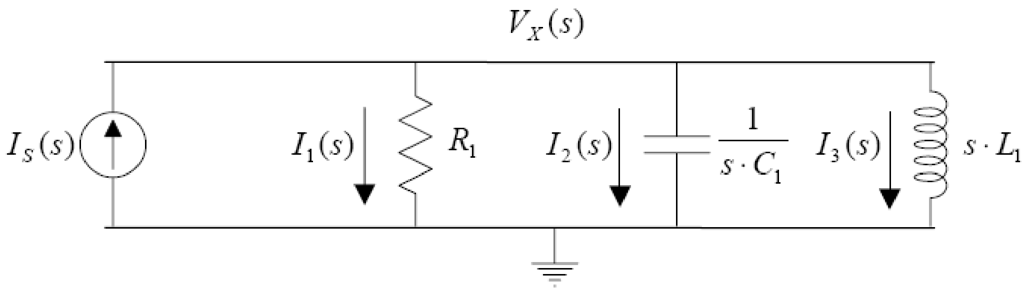
6. Reference

Original Notes from Meghdad Hajimorad (“Amin”) for EE 105. Year 2004

Last Modified by Bill Hung for EE 40. 14 August 2006

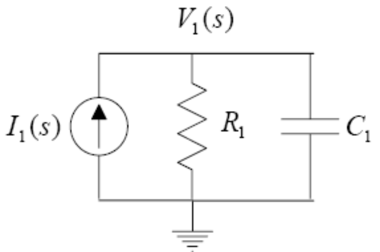
Other Things You Might Want to Remember for the Final

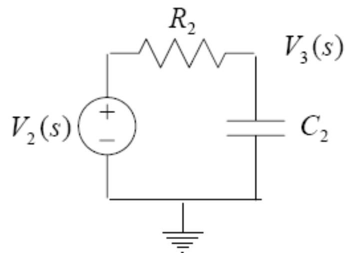
1. Impedence for Capacitor and Inductor



2. Common Transfer Functions

These two circuits depicted appear frequently in circuit analysis/design. Hence, it is useful to have their associated transfer functions memorized. It can be shown that





$$\frac{V_1(s)}{I_1(s)} = \frac{R_1}{1 + s \cdot R_1 \cdot C_1}$$

$$\frac{V_3(s)}{V_2(s)} = \frac{1}{1 + s \cdot R_2 \cdot C_2}$$

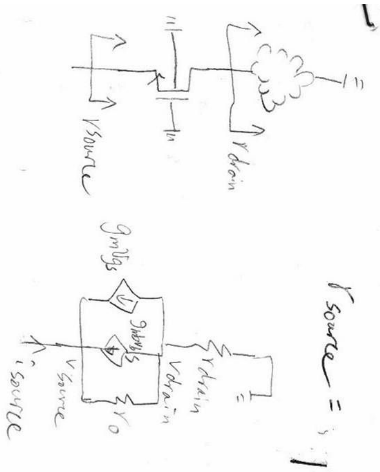
3. NMOS I_{DS} in Triode Region and Saturation Region

$$Triode/Linear: \quad I_{DS} = \mu_n C_{ox} (\frac{W}{L})_n \cdot (V_{GS} - V_{Tn} - \frac{V_{DS}}{2}) \cdot V_{DS}$$

$$Saturation: \quad V_{GS} > V_{Tn} \quad and \quad V_{DS} > V_{GS} - V_{Tn}$$

$$Saturation: \quad I_{DS} = \frac{\mu_n C_{ox}}{2} \cdot (\frac{W}{L})_n \cdot (V_{GS} - V_{Tn})^2 \cdot (1 + \lambda_n V_{DS})$$

Derivation for r_{source} and r_{drain} (with Vbs). You do not need to know this.



$$r_{source} = \frac{r_o + r_{drain}}{1 + (g_m + g_{mb}) r_o} \approx \frac{r_o + r_{drain}}{(g_m + g_{mb}) r_o}$$

$$r_{source} = \frac{v_{source}}{i_{source}}$$

$$-g_m v_{gs} - g_{mb} v_{bs} + \frac{v_{source} - v_{drain}}{r_o} - i_{source} = 0$$

$$-g_m(-v_{source}) - g_{mb}(-v_{source}) + \frac{v_{source} - i_{source} r_{drain}}{r_o} - i_{source} = 0$$

$$v_{source}\left(g_m + g_{mb} + \frac{1}{r_o}\right) = i_{source}\left(\frac{r_{drain}}{r_o} + 1\right)$$

$$\frac{v_{source}}{i_{source}} = \frac{\frac{r_{drain}}{r_o} + 1}{g_m + g_{mb} + \frac{1}{r_o}}$$

$$r_{source} = \frac{r_{drain} + r_o}{(g_m + g_{mb}) r_o + 1}$$

$$g_m v_{gs} + g_{mb} v_{bs} + \frac{v_{drain} - v_{source}}{r_o} + \frac{v_{drain}}{r_{drain}} = 0$$

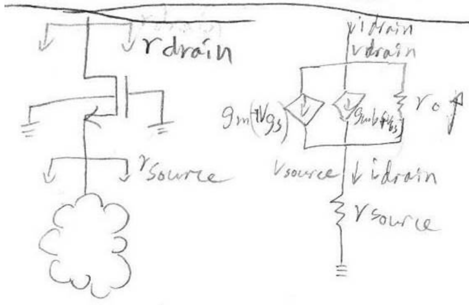
$$-v_{source}(g_m + g_{mb}) + \frac{i_{source} r_{drain} - v_{source}}{r_o} + i_{source} = 0$$

$$i_{source}\left(\frac{r_{drain}}{r_o} + 1\right) - v_{source}\left(+(g_m + g_{mb}) + \frac{1}{r_o}\right) = 0$$

$$\frac{\frac{r_{drain}}{r_o} + 1}{(g_m + g_{mb}) + \frac{1}{r_o}} = \frac{v_{source}}{i_{source}}$$

$$r_{source} = \frac{r_{drain} + r_o}{(g_m + g_{mb}) r_o + 1}$$

$$r_{drain} = r_o + r_{source} + (g_m + g_{mb}) r_o \cdot r_{source}$$



$$r_{drain} = \frac{v_{drain}}{i_{drain}}$$

$$-g_m(+v_{gs}) - g_{mb}(+V_{bs}) + \frac{v_{source} - v_{drain}}{r_o} + i_{drain} = 0$$

$$-g_m v_{gs} - g_{mb} v_{bs} - \frac{v_{drain}}{r_o} + \frac{i_{drain} \cdot r_{source}}{r_o} + i_{drain} = 0$$

$$g_m(+i_{drain} r_{source}) + g_{mb}(+i_{drain} \cdot r_{source}) - \frac{v_{drain}}{r_o} + \frac{i_{drain} \cdot r_{source}}{r_o} + i_{drain} = 0$$

$$i_{drain}\left(+g_m r_{source} + g_{mb} r_{source} + \frac{r_{source}}{r_o} + 1\right) - \frac{v_{drain}}{r_o} = 0$$

$$r_o\left(+g_m r_{source} + g_{mb} r_{source} + \frac{r_{source}}{r_o} + 1\right) = +\frac{v_{drain}}{i_{drain}}$$

$$r_{drain} = (+g_m r_{source} r_o + g_{mb} r_{source} r_o) + r_{source} + r_o$$

$$r_{drain} = +(g_m + g_{mb}) r_{source} r_o + r_{source} + r_o$$