Lab 23 Digital Filtering

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Lab Time: 9-12pm Wednesday
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Station 8

4. Questions

4.1 What value of α did you use for your low-pass digital filter?

The definition of α is

α = e^(-Δt/RC) [Derenzo, 474]

Δt = 1/fs = 1/40Mhz = 50us , where Δt is the filter process time
RC = 51k ohm * 0.1uF, where R is the resistor and C is the capacitor used for the analog filter.

Therefore
α = e^[-50us/(51k*0.1u)]
α =0.990

Notice the resistor (R) and capacitor (C) values used in the lab slightly differ from the values above. For the calculations here, ideal values were used.

4.2 What value of α would you use in this exercise if your sampling frequency were 100 kHz?
Δt = 1/fs = 10us,
RC = 51k ohm * 0.1uF
α = e^(-Δt/RC)
α = e^[-10us/(51k ohm * 0.1uF)]
α = 0.998

4.3 Compute the impulse responses for the following filters:

a. $y_i = x_{i-1} + 0.5\, y_{i-1} + 2\, y_{i-2}$
b. $y_i = x_{i-1} + 0.9\, y_{i-4}$
c. $y_i = x_{i-1} - y_{i-1}$
Hint: Use $x_0 = 1 \quad x_{\neq 0} = 0$

a.
For the equation

$$y_i = x_{i-1} + 0.5\, y_{i-1} + 2\, y_{i-2}$$

If we list out the numbers assuming $x_0 = 1 \quad x_{\neq 0} = 0$, we have
y0 =0
y1 = x0 + 0.5*y0 + 2*y_{-1} = 1 + 0 + 0 = 1
y2 = x1 + 0.5*y1 + 2*y0 = 0 + 0.5*1 + 0 = 0.5
y3 = x2 + 0.5*y2 + 2*y1 = 0 + 0.5*0.5 + 2*1 = 2.25
y4 = x3 + 0.5*y3 + 2*y2 = 0 + 0.5*2.25 + 2*0.5 = 2.125
y5 = x4 + 0.5*y4 + 2*y3 = 0 + 0.5*2.125 + 2*2.25 =5.5625
y6 = x5 + 0.5*y5 + 2*y4 = 0 + 0.5*5.5625 + 2*2.125 = 7.03125
…

In general, we can solve this second-order linear homogeneous recurrence relations (a differential equation) by the distincts-roots theorem

Initial conditions y0=0, y1=1, so for i>1,
Suppose Yi=r^i
ri=0.5*r^i-1 + 2*r^(i-2)
r^(i-2)*(r^2-0.5*r-2) = 0
r = [0.5+ sqrt(1/4 + 4*2)]/2 or [0.5- sqrt(1/4 + 4*2)]/2
r = 1.6861 or -1.1861

In general, the solution will have the form of Yi= C*r^i + D*s^i, where r and s are the roots of the characteristic equation. Here r=1.6861 and s=-1.1861.

Yi = C*1.6861^i + D*-1.1861^i

Substituting the initial conditions
Y0=0 =C+D
Y1=1= C*1.6861 + D*-1.1861
C= 0.34817, D=-0.34817
so
Yi = 0.34817*1.6861^i + -0.34817*-1.1861^i

b.
For the equation
$y_i = x_{i-1} + 0.9\ y_{i-4}$
If we list out the numbers assuming $x_0 = 1$ $x_{\neq 0} = 0$, we have
y0 =0
y1=1, y2=0, y3=0, y4=0,
y5=0.9*1, y6=0, y7=0, y8=0,
y9=0.9^2, y10=0, y11=0, y12=0,
..
In general,
y(i<1) = 0 and y(1+4i) = 0.9^i

c.
For the equation
$y_i = x_{i-1} – y_{i-1}$
If we list out the numbers assuming $x_0 = 1$ $x_{\neq 0} = 0$, we have
y0=0
y1=1-0=1
y2=0-1=-1
y3=0-(-1) = 1
y4= 0 – (1) = -1

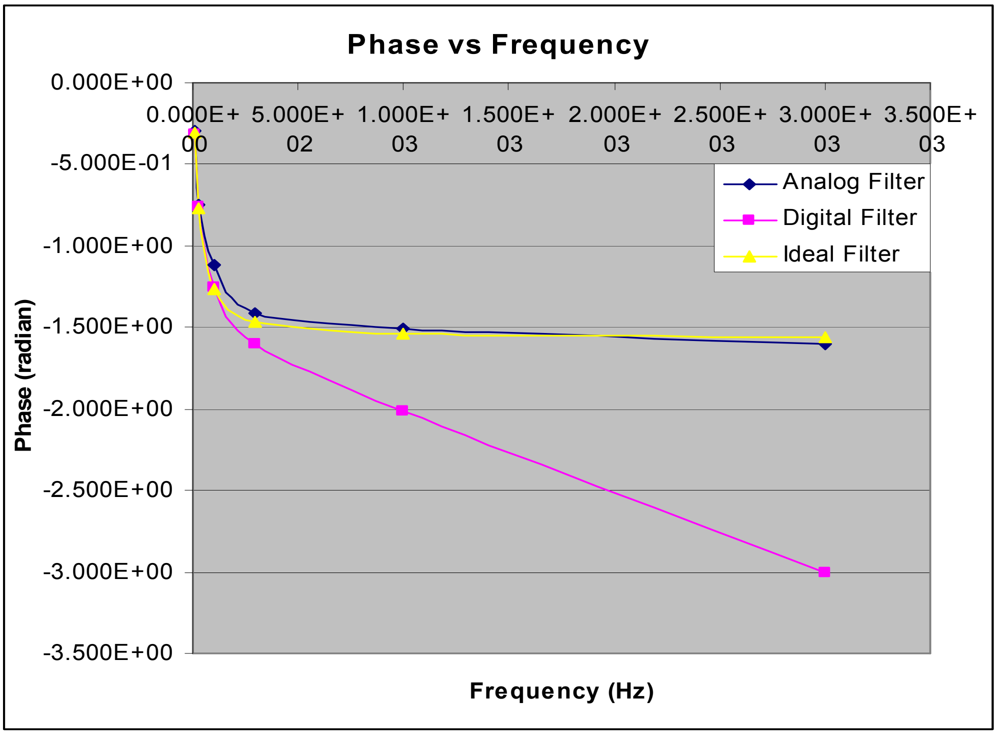
In general,
y(i<1) =0
y(2i) = -1
y(2i-1) = 1

4.4 At what frequency did the digital filter have a phase shift of 90°? At this frequency, what was the phase shift of the analog filter?

Table 1. Phase for both filters and the ideal low-pass filter

Frequency (Hz)	Ideal Φ (radian)	Analog filter Φ (radian)	Digital filter Φ (radian)
1.000E+01	-3.101E-01	-2.957E-01	-3.173E-01
3.000E+01	-7.657E-01	-7.524E-01	-7.547E-01
1.000E+02	-1.268E+00	-1.120E+00	-1.257E+00
3.000E+02	-1.467E+00	-1.414E+00	-1.606E+00
1.000E+03	-1.540E+00	-1.508E+00	-2.011E+00
3.000E+03	-1.560E+00	-1.602E+00	-3.000E+00

A Plot for the data from Table 1



A phase shift of 90 degrees equals to pi/2=1.57 rad. Based on the data from Table 1, a phase shift of 1.57 rad happened between frequency 100Hz and 300Hz. The frequency is estimated by the weighted average below (assuming the ratio is linear):

Frequency (phase shift =90 degrees)
= 100 + (300-100)/(1.606-1.257) * (1.57-1.257)
= 180.63Hz

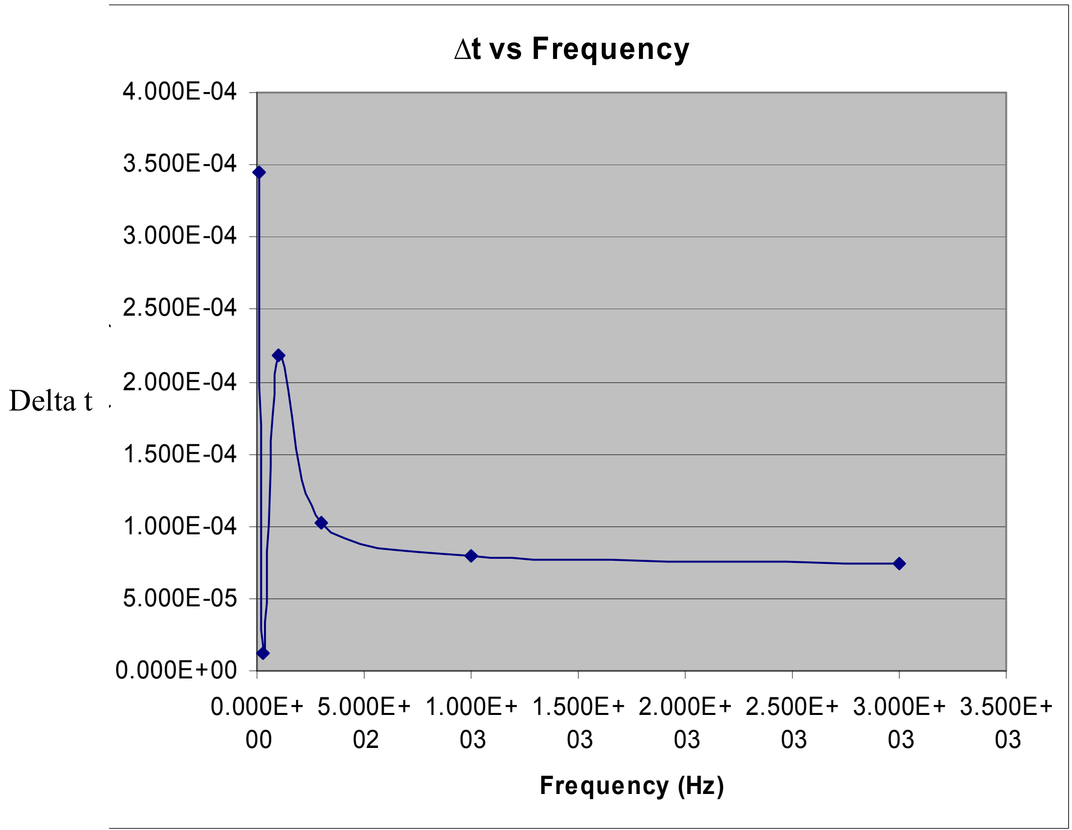
Analog Phase Shift at frequency 180.63Hz is about
= 1.12 + (1.414-1.12) /(300-100) * (180.3-100)
= 1.238 rad
This number is an approximation because the data is not exactly linear.

4.5 Was the difference in time shift Δt between the two filters independent of frequency? What did you expect?

The Δt should be a constant, but it is dependent of input frequency. According to the plot in section 4.4, the phase increases as the input frequency increases.

$\Delta t = \Delta\Phi/(2*pi*f)$ where f is the input frequency.

Since Δt is constant, as input frequency f increases, Φ increases. This is shown in the plot in section 4.4.



The experimental result agrees with my expectation. The plot is not a straight line. As the plot of delta t versus frequency indicates, the time shift (delta t) is not independent of the input frequency.