# Lab 21 Fast Fourier Transforms of Sampled Data

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**4.1 How would you expect the last 64 values of the magnitude (i.e. $F_{513}$ -> $F_{1023}$) to compare with the first 64 values after the dc term (i.e. $F_1$ -> $F_{512}$)? Give an explicit relationship.**

The 64 values of the magnitude of Fn should have the same magnitude as the 64 of the $F_{1023-n}$. This is because from the equations.

$$H_{M+n} = \sum_{k=0}^{M-1} h_k e^{-j2\pi k} e^{-j2\pi kn/M} = \sum_{k=0}^{M-1} h_k e^{-j2\pi kn/M} = H_n$$

Between $M/2$ and $M$ samples per $S$, we have the following r

$$H_{M-n} = \sum_{k=0}^{M-1} h_k e^{-j2\pi k} e^{+j2\pi kn/M} = \sum_{k=0}^{M-1} h_k e^{+j2\pi kn/M}$$

[Derenzo, 403]

Hn and $H_{M-n}$ (where M is 1024 in this case) only differs from the sign of the exponential term. Changing the sign of the exponential term in the equations above is equivalent to taking the conjugate of the function. In other words, $H_{1024-n} = H_n$*, where * denotes the conjugate. Therefore, from the equations, I would have expected the magnitude on the left will be "mirrored" on the right side of the plot. That is to say I would expect the magnitude of Fn to be the same as the magnitude of $F_{1024-n}$.

**4.2 For the FFT of the sine wave, did you observe any nonzero amplitudes at integral multiples of the fundamental frequency? What could cause such nonzero Fourier coefficients?**

For the Fast Fourier Transform (FFT) of the sine wave, which is 6.5 cycles for 1024 samples, I observed nonzero amplitudes at integral multiples of the fundamental frequency. The harmonics (integral multiples of the fundamental frequency) have non-zero amplitudes because during sampling, data is collected for a finite period of time. In the time domain, the input signal is multiplied with a "sampling box" from time –S/2 to +S/2 (where S is the sampling time). The sampled signal will only have data from –S/2 to +S/2 and zero everywhere else.
In the frequency domain, the sampled signal will be the convolution of the input signal and the "sampling box". The input signal is 6.5 cycles, which is 126.9Hz. Therefore the frequency plot of the input signal should peak at 126.9 kHz and (20kHz-126.9 kHz). The frequency plot of the "sampling box" is a sinc function. Since multiplying in time domain equals convolving in frequency domain. By convolving the input signal and the sinc function (of the sampling box) in the frequency domain, we have those non-zero amplitudes for the harmonics because of the non-integral number of cycles (6.5 cycles).

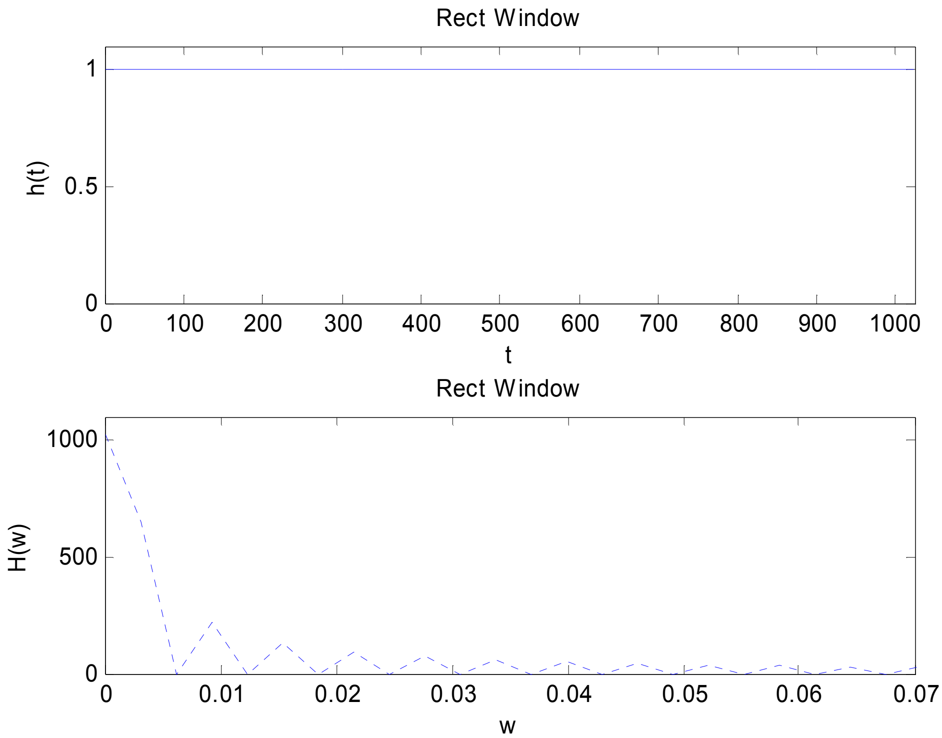


**Figure 1**

As shown in figure 1, the frequency plot does not have zero crossing at integral multiples of the fundamental frequency.
Ideally, if the input signal has an integral number of cycles, the harmonics would be zero because the sinc function has zero-crossing at the harmonics. However, since the input signal has non-integral number of cycles (6.5 cycles) there are nonzero amplitudes at the harmonics.

**4.3 Since the data-acquisition loop was not synchronized with the output of the wave generator, would you expect the real and imaginary components of the Fourier amplitude to be the same if the exercise was repeated?**

No, the real and imaginary components of the Fourier amplitude will not be the same if the exercise was repeated. This is because there will be a phase difference when the data is sampled again. The phase different will introduce a phase term in the signal. For example if the signal for the first time was cos(2*pi*f) + i*sin(2*pi*f), the second time the signal might be cos(2*pi*f + phase) + i*sin(2*pi*f + phase). The phase term would change the real and imaginary components of the Fourier amplitude.

Although the real and imaginary component of the Fourier amplitude is different, the Fourier amplitude still has the same value if the exercise is repeated again.

**4.4 What benefit did the Hann window have for sampling and Fourier transforming a noninteger number of cycles?**

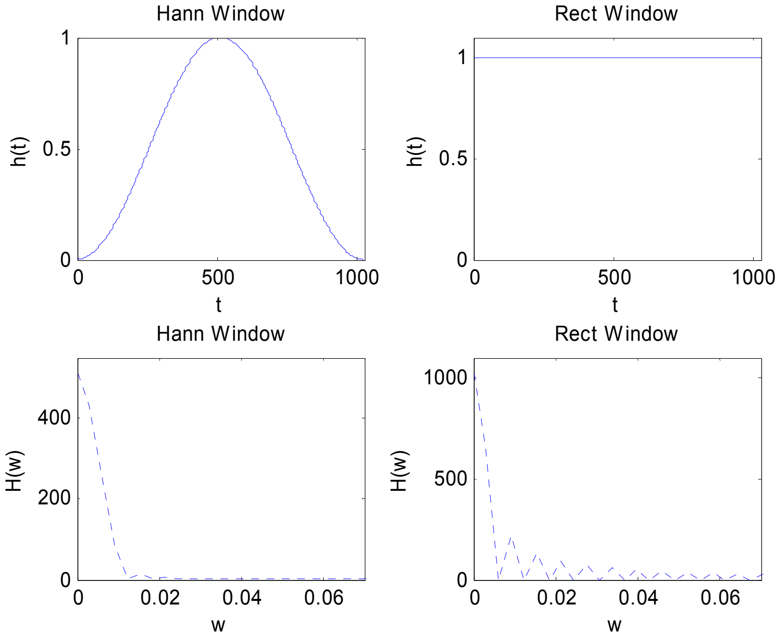


**Figure 2**

When the input signal has frequency components that are not integral multiple of the sampling frequency, the convolution will cause the frequency amplitude to spread out over a range of frequencies These unwanted spreading is referred to as "spectral leakage" [Derenzo, 409].

By applying a Hann window, the time signal of the input gradually tapers off so the signal at t=-S/2 and t=S/2. This will reduces the spreading of amplitudes, and gives a more distinct frequency for the periodic input waveforms. The resulting frequency plot will reduce the "wave" of unwanted frequency. Figure 2 shows the effect of applying the Hann window (left) versus the effect of applying a rectangular window (right).

**4.5 What benefit did the anti-aliasing filter have for sampling and Fourier transforming the square wave?**

Let's take M=128 and a 5 cycle square wave as an example. For an input signals, there are integral multiples of the fundamental frequency called the harmonics. In the frequency plot, we have all the harmonics. Since $H_n = H_{M-n}^*$, $F_n=F_M-n$ where M is the number of sample points. At the thirteenth harmonic, Fourier indices at 13 x 5 = 65 and also at 128-13x5=63. The 63, which is supposed to be high frequency, crosses over to the lower frequency and caused aliasing. Similar result happens for higher harmonics like the fifteenth harmonic, the seventeenth harmonics, and so on.

In order to prevent the aliasing, the anti-aliasing filter can be used to filter out any frequency higher than half of the sampling frequency. An anti-aliasing filter is a low pass filter, which filters out frequency higher than half of the sampling frequency. After filtering, the high frequency was eliminated and would no longer cause aliasing to the lower frequency.
In practical anti-aliasing filter, the cut-off frequency is not set at exactly half of the sampling frequency because the cut-off frequency of the filter is not "sharp" enough.