C.9 Plucked String Instrument Solutions

C.9.1 In-lab section

1. The moving average is an LTI system, so if the input is $x(n) = e^{j\omega n}$, then the output is $H(\omega)e^{i\omega n}$, where H is the frequency response. We can determine the frequency response using this fact by plugging this input and output into the difference equation, getting

$$H(\omega)e^{i\omega n} = 0.5(e^{i\omega n} + e^{i\omega(n-1)}) = 0.5e^{i\omega n}(1 + e^{-i\omega}).$$

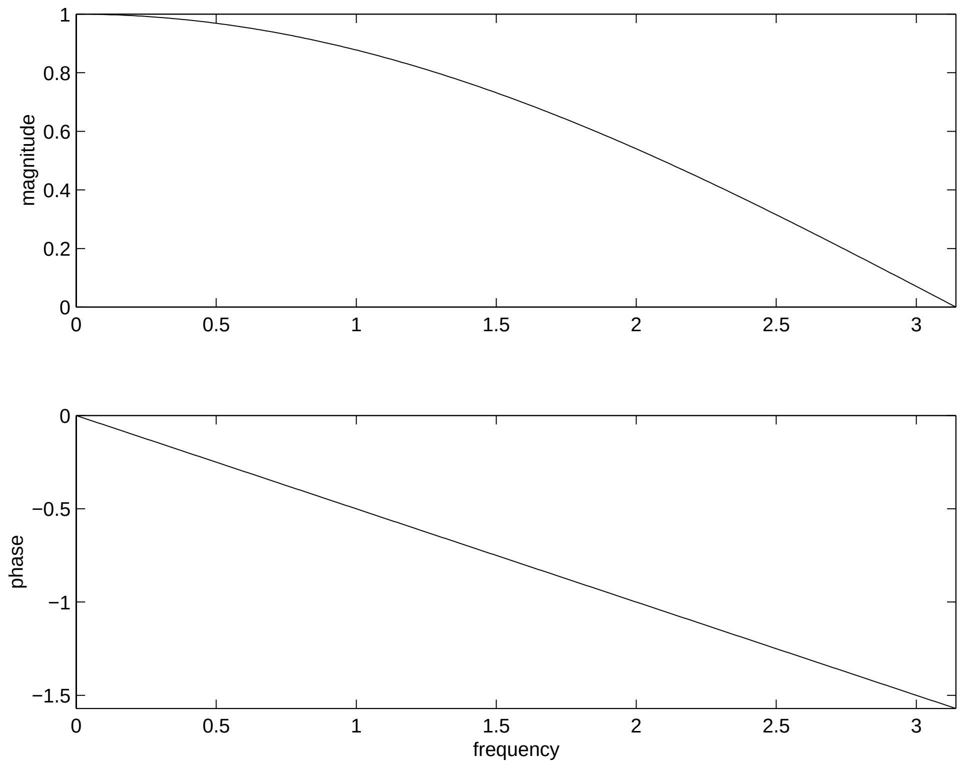
Eliminating $e^{i\omega n}$ on both sides we get

$$H(\omega) = 0.5(1 + e^{-i\omega}).$$

We can plot the magnitude and phase of this using Matlab as follows:

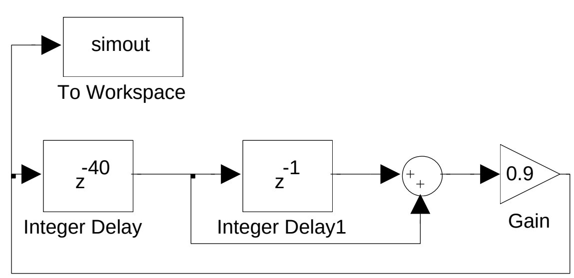
```
omega = 0:pi/200:pi;
H = 0.5*(1+exp(-i*omega));
subplot(2,1,1);
plot(omega, abs(H));
axis([0, pi, 0, 1]);
ylabel('magnitude');
subplot(2,1,2);
plot(omega, angle(H));
axis([0, pi, -pi/2, 0]);
ylabel('phase');
xlabel('frequency');
```

This produces the following plots:



Notice that the phase is indeed linear, and the slope is $-1/2$, which suggests that the delay is $1/2$ sample.

2. A modification of the comb filter model is given below:



The Gain block parameter is set to 0.99*0.5 to account for the 0.5 factor in the lowpass filter. The lowpass filter itself is realized with a unit delay and a summer. As with the comb filter model from the previous lab, the 40 sample delay is initialized with random numbers using

```
randn(1,40)
```

The resulting sound is dramatically more guitar-like.

3. The fundamental frequency with $N = 40$ plus the half-sample delay of the lowpass filter is $8000/40.5 = 197.5$ Hz.

All of the achievable frequencies are of the form $8000/(N + 0.5)$ for integers N. We can compute a range of these in Matlab as follows:

```
>> N = [15:25];
>> 8000./(N+0.5)

ans =

  Columns 1 through 7

  516.1290  484.8485  457.1429  432.4324  410.2564  390.2439  372.0930

  Columns 8 through 11

  355.5556  340.4255  326.5306  313.7255
```

From this, we see that 440 is not achievable. It is flanked by 457 above and 432 below. These are both quite far off, and would not be acceptable for musical purposes.