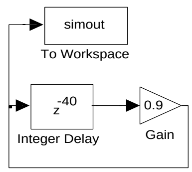

Figure C.24: Comb filter with no input, but random initial values in the delay instead.

C.8.2 Independent section

1. If the input is

$$x(n) = e^{i\omega n}$$

then the output is

$$y(n) = H(\omega)e^{i\omega n}.$$

We can substitute these into the original LCCDE,

$$y(n) = x(n) + \alpha y(n - N)$$

to get

$$\begin{aligned} H(\omega)e^{i\omega n} &= e^{i\omega n} + \alpha H(\omega)e^{i\omega(n-N)} \\ &= e^{i\omega n}(1 + \alpha H(\omega)e^{-i\omega N}). \end{aligned}$$

Eliminating $e^{i\omega n}$ on both sides we get

$$H(\omega) = 1 + \alpha H(\omega)e^{-i\omega N}.$$

Solving for $H(\omega)$ we get

$$H(\omega) = \frac{1}{1 - \alpha e^{-i\omega N}}.$$

To plot the magnitude of this over 0 to 4 kHz, note that ω varies from 0 to $2\pi \times 4000/8000 = \pi$. We can choose how many samples of the frequency response we wish to plot. Choosing 500, the following Matlab code constructs the plot:

```
omega = 0:pi/500:pi;
alpha = 0.99;
N = 40;
magnitude = abs(1./(1-alpha*exp(-i*omega*N)));
plot(omega, magnitude);
xlabel('frequency');
ylabel('amplitude');
axis([0, pi, 0, 110]);
```

This produces the plot shown in figure C.25.

The frequency response indicates that the output will have a fundamental of 200 Hz ($2\pi \times 200/8000 = 0.1571$ radians) plus harmonics at multiples of 200 Hz. This is why the output sounds like a 200 Hz tone, albeit richer than a sinusoidal tone.

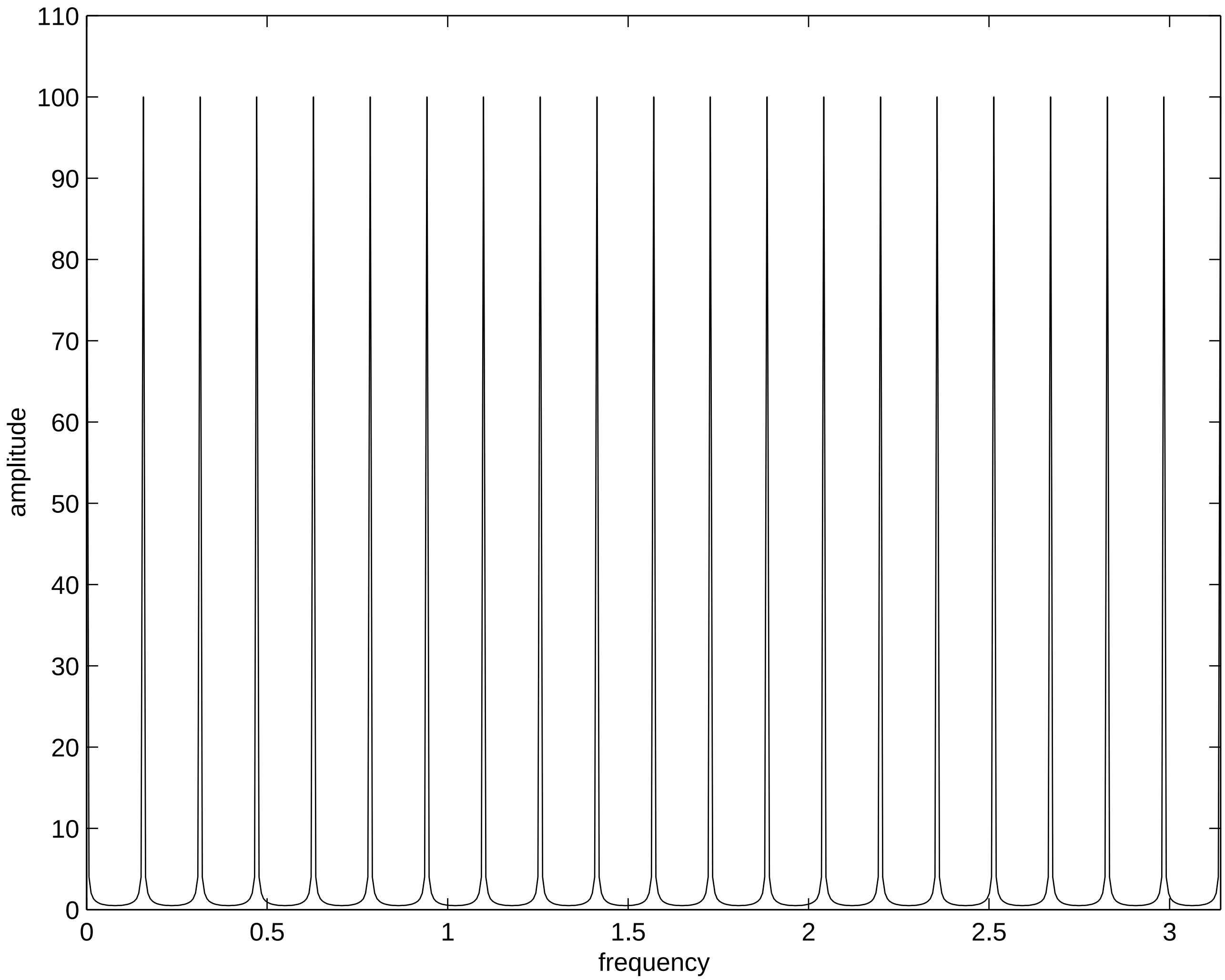

Figure C.25: Frequency response of the comb filter.