C.5.2 Independent section

1. The form of the desired response is the same as that in problem 3 of the in-lab section, so A, b, c, and d are given there. The impulse response is given by

$$h(n) = \begin{cases} d, & \text{if } n = 0 \\ c^T A^{n-1} b, & \text{if } n \geq 1 \end{cases}$$

which for the specified values of A, b, c, and d can be written (using the results of exercise as suggested in the hint)

$$h(n) = \sigma^n \sin(\omega n).$$

To get the desired impulse response, we simply set

$$\omega = 2\pi 440/8000$$

(which has units of radians per sample), and

$$\sigma = \exp(-5/8000).$$

We can use the same function defined in the in-lab section. To get 8000 samples of the impulse response and play it as a sound we simply do

```
x = [1, zeros(1,7999)];
sound(boing(x, exp(-5/8000), 2*pi*440/8000));
```

2. To construct the input, we simply do:

```
x = repmat([1, zeros(1,1599)], 1, 10);
sound(boing(x, exp(-5/8000), 2*pi*440/8000));
```

3. In lab C.1 we calculated one second of sound using the Matlab script

```
t = 0:1/8000:1;
n = exp(-5*t).*sin(2*pi*440*t);
sound(n, 8000);
```

We will count only the operations in the middle line, and we will assume that 8000, rather than 8001 samples are computed. Each sample requires one evaluation of `exp` and one evaluation of `sin`, for a total of 40 multiplications and 30 additions. Each argument requires one multiplication, and the results are multiplied, so there are 43 multiplications per sample. For 8000 samples, that implies $8000 \times 43 = 344000$ multiplications and $8000 \times 30 = 240000$ additions.

For the state machine model, examining the `boing` function, we see that each output sample involves the calculations given by

```
y(i) = c'*s;
s = A*s + b*x(i);
```

The first line is two multiplications and one addition. The second line is 6 multiplications and two additions. The total is therefore 8 multiplications and 3 additions per sample. For 8000 samples, that implies $8000 \times 8 = 64000$ multiplications and $8000 \times 3 = 24000$ additions. The state machine realization is considerably less expensive by this measure.